THE FAN:

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THEORY AND PRACTICE OF CENTRIFUGAL AND AXIAL FANS.

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BY

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PREFACE.

In the following pages I give a theory of the fan which differs considerably from anything that I have seen in print, and which may therefore meet with some criticism. In all works in this or any other language that I have read I have found the equation:

$$H = \frac{k c_1^2}{1 + \frac{o^2}{a^2}},$$

where H is the head of air against which the fan works, c_1 is the tip speed, a is a constant for any particular fan, and o is the equivalent orifice.

That this equation cannot be that of all fans is obvious when we remember that in many fans the manometric efficiency increases at first as the orifice increases from zero, while this equation states that it decreases; nor can it be maintained that the equation applies to fans whose manometric efficiency is greatest at zero orifice, for if the curve of manometric efficiency be drawn with this as ordinate and orifices as abscissæ, it will be found that the tangent at the point where the curve cuts the vertical axis is horizontal; and that this is not the case may be seen from fig. 44, which gives curves for eleven fans differing widely in construction. Having already studied the centrifugal pump, it occurred to me about ten years ago that its theory might be applied to that of the fan; and except that the fan does not actually lift air, as the pump lifts water, but acts like a centrifugal pump that pumps against the resistance of horizontal piping only, I consider that the same theory may be applied to both.

In Chapter XII. I have endeavoured to show that my theory agrees with the results of experiment, as far as these may be trusted. The following pages commence with the theory of the centrifugal fan, following which are experiments with and descriptions of this type; and in Chapter XIV. is given a description of Prof. Rateau's high-pressure fans, in whose design it may be mentioned the variation of the density of the air must be taken into account. In Chapter XV. will be found an imperfect theory of propeller fans, imperfect because I cannot find all the information I require from published experiments. Following this will be found descriptions of this type and of Prof. Rateau's screw fans, the theory of which closes the book.

I hope this book will be of service to those who have to design, or who wish to understand the working of fans. There are many very inferior fans largely used, which do their work very wastefully, and it would be satisfactory to see these replaced by others of scientific design.

CHAS. H. INNES.

Rutherford College,
Newcastle-on-Tyne,
October, 1904.

ERRATUM.

CHAPTER XIV. (p. 182).—It should be explained that V and δ in the following figures—86, 87, 92—are the same, and also that $M = \mu$; also for volumetric efficiency

$$V = \frac{Q}{c_1 r_1^2} \text{ read } V = \frac{Q}{c_1 r_1^2} = \delta;$$

and for manometric efficiency

$$M = \frac{g H}{c_1^2} = \mu.$$

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CHAPTER I.

1. *The Conservation of Energy.*—Energy is indestructible, although it may appear in a number of forms, some of which are useful to man, while others are not. Thus steam at a high pressure contains energy in the form of heat, part of which can be converted into useful work while the remainder is wasted in overcoming friction, or is rejected with the condensed steam at a low temperature, and is of no service to man. But the law of conservation of energy supplies us with equations which are of the utmost service in correctly designing machines in which a flow of fluids takes place, because we know that changes of pressure, volume, and velocity are accompanied by alterations of the forms in which the initial energy existed, but that the quantity of energy is unaltered. Although in this article we are dealing with air, a compressible gas, and should therefore, in strict accuracy, take into account the alteration of volume that accompanies change of pressure, yet since this change of pressure is so small, it is not necessary to do so, especially if we consider the volume passing through the fan to be the volume occupied by the air at a pressure which is the arithmetic mean between that at suction and discharge. Let P be the difference of the pressures per square foot, and Q the mean volume in cubic feet per second of the air passing through the fan measured as above; then P Q is the useful work done per second, supposing of course that the manometer is placed in a position where the velocity of the air is not great, or has a bend at the end turned towards the current of air so that it forms a sort of Pitot tube.

2. *The Relation between H, h, and v.*—Suppose a liquid whose density is D lbs. per cubic foot contained in a vessel, fig. 1, whose free horizontal surface is at a height H above any assumed level and that the surface is maintained at this level as the liquid flows down a pipe connected to the vessel

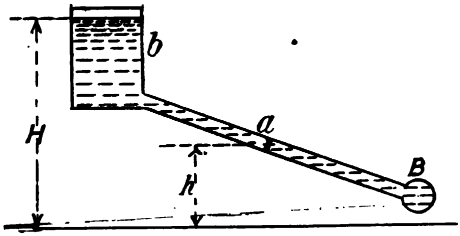


FIG. 1.

below the surface, and let us at first neglect all losses due to friction; then, if at a point a at a height h above the given level the liquid has a velocity of v feet per second and a pressure of p lbs. per square foot, we shall show that

$$H = h + \frac{p}{D} + \frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For suppose a piston at a moving down at a velocity v, then if A is the section of the pipe in square feet when it has moved 1 ft., the pressure above the piston has done p A foot-pounds of work, at the expense of A cubic feet of liquid, or p foot-pounds at the expense of each cubic foot, or, as one cubic foot contains D lbs., $\frac{p}{D}$ foot-pounds of work are done at the expense of each pound of liquid. Thus the H foot-pounds of potential energy of each pound at b appear in three different forms at a, viz., potential energy h, kinetic energy $\frac{v^2}{2g}$, and energy due to pressure, or pressure energy $\frac{p}{D}$. Hence we say that a liquid under pressure p, velocity

v, and at a height h, has a pressure head $\frac{p}{D}$, velocity head $\frac{v^2}{2g}$, and actual head h, while the equivalent head due to all three is H.

3. *Reasons for Treating the Air as if it had a constant volume.*—The highest water gauge found in practice is 12 in., and as the water barometer is 34 ft. and air contains a considerable quantity of moisture, and we may therefore consider the compression to be isothermal, the greatest compression is $\frac{1}{34}$th of the original volume, and if we measure the discharge by the mean volume, the actual volume at any instant cannot be greater or less than this by $\frac{1}{68}$th part of the mean volume. Hence the work produced by expansion, or required for compression may be entirely neglected, and we may treat the air as if it were a liquid, obeying equation (1).

Suppose, for example, that air weighs ·075 lb. per cubic foot, and moves with a velocity of 40 ft. per second in a straight pipe which gradually enlarges in section so that the velocity is reduced to 20 ft. per second, what will the change of pressure per square foot be if the pipe is horizontal? Let p_1, v_1 be the pressure and velocity when the latter is 40 ft. per second, and p_2, v_2 similar quantities when it is 20. Then,

$$\frac{p_2 - p_1}{D} = \frac{v_1^2 - v_2^2}{2g} = \text{change of pressure head}$$

$$p_2 - p_1 = \frac{\cdot 075}{64}(1600 - 400)$$

$$= 1\tfrac{13}{32} \text{ lb. per square foot}$$

$$= 18\tfrac{3}{4} \text{ ft. change of pressure head.}$$

It will be seen later that the mechanical efficiency of a fan can be considerably increased by using various means for gradually reducing the velocity of the air, and thus increasing its pressure.

4. *Losses of Head by Surface Friction and Change of Velocity.*—When fluids flow in passages, energy may be

wasted by surface friction, by bends and corners, and by sudden changes of velocity and direction. The loss of energy per pound is called the loss of head. The total force necessary to overcome surface friction in a passage of a given material is proportional to the product of the surface in square feet, and the square of the velocity of the air, so that if F = force, and A the area

$$F = \frac{m A v^2}{2 g}$$

where m is a coefficient depending on the fluid and the roughness of the surface, let a be the section of the passage where v is the velocity, then

$$\frac{F}{a} = \text{pressure per square foot}$$

$$= \frac{m A v^2}{2 a g}$$

and if D = weight of fluid per cubic foot the loss of head due to friction is

$$h_1 = \frac{F}{a D} = \frac{m A v^2}{2 a D g}.$$

If the passage is a pipe of circular section of diameter d feet, length l

$$h_1 = \frac{4 m l v^2}{d D 2 g} = \zeta \frac{4 l}{d} \frac{v^2}{2 g} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If $$\zeta \frac{4 l}{d} = f,$$

then f is the coefficient of resistance of the pipe referred to the velocity v. And as the losses of head due to bends and elbows, and passages of any form are proportional to the square of the velocity, in a given machine we can say that the loss of head is

$$F_1 \frac{v^2}{2 g}$$

in addition to that part of the loss of head due to sudden changes of direction and velocity, where F_1 is the coefficient of resistance of the machine referred to the velocity v.

Professor Unwin in "The Development and Transmission of Power," gives

$$\zeta = \cdot 0027 \left(1 + \frac{3}{10d}\right)$$

where d is in feet. If the pipe is not of circular section formula (2) becomes

$$h_1 = \zeta \frac{l}{m_1} \frac{v^2}{2g}$$

where m_1 is the mean hydraulic depth of the pipe—*i.e.*, section divided by circumference.

M. Lelong* gives $\zeta = \cdot 006$.

When a sudden change of direction of motion and of velocity takes place, such as that represented in fig. 2 from

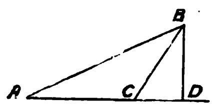


FIG. 2.

A B to A C where A B represents a velocity v, A C a velocity v_1, and the angle B A C is called θ, then the loss of head

$$h_2 = \frac{BC^2}{2g}$$

$$= \frac{v^2 + v_1^2 - 2 v v_1 \cos \theta}{2g} \quad . \quad . \quad . \quad . \quad (3)$$

and obviously has its least value when B C coincides with B D, the perpendicular on A C, so that

$$v_1 = v \cos \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

* Du Calcul des Ventilateurs, by M. Lelong.

Let p be the pressure before the change and p_1 after, then

$$\frac{p_1}{D} + \frac{v_1^2}{2g} + \text{loss of head} = \frac{p}{D} + \frac{v^2}{2g}$$

$$\frac{p_1 - p}{D} = \text{gain of pressure head}$$

$$= \frac{v^2 - v_1^2}{2g} - \frac{v^2 + v_1^2 - 2\,v\,v_1 \cos\theta}{2g}$$

$$= \frac{v\,v_1 \cos\theta - v_1^2}{g} = \frac{A\,C\,C\,D}{g} \quad . \quad . \quad . \quad . \quad (5)$$

which obviously has a maximum value when A C = C D, so that where a sudden change of direction must take place, and it is intended afterwards to convert a large part of the velocity head into pressure head by a gradually expanding pipe, v_1 should equal $v \cos\theta$, but where no expanding pipe can be fitted A C must be $\frac{1}{2}$ C D, and therefore

$$v_1 = \tfrac{1}{2}\,v \cos\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

When a sudden reduction of velocity takes place from v to v_1 without change of direction the loss of head is

$$h_3 = \frac{(v - v_1)^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and therefore the gain of pressure head is obtained as follows :—

$$\frac{p_1 - p}{D} = \text{gain of pressure head,}$$

$$= \frac{v^2 - v_1^2}{2g} - \text{loss of head,}$$

$$= \frac{v^2 - v_1^2}{2g} - \frac{(v - v_1)^2}{2g} = \frac{v\,v_1 - v_1^2}{g} \quad . \quad (8)$$

The formulæ for losses of head at bends are

$$h_4 = \left\{ 0{\cdot}131 + 1{\cdot}847\left(\frac{d}{2\,C}\right)^{\frac{7}{2}} \right\} \frac{\phi}{180}\,\frac{v^2}{2g} \quad . \quad . \quad (8a)$$

for bends of pipes with circular section diameter d ft.

$$h_5 = \left\{ 0{\cdot}124 + 3{\cdot}104\left(\frac{d}{2\,C}\right)^{\frac{7}{2}} \right\} \frac{\phi}{180}\,\frac{v^2}{2\,g} \quad . \quad . \quad (8b)$$

for pipes of rectangular section of height d; C being the mean radius of bend in each case and ϕ the angle of bend. These are of doubtful accuracy.

CHAPTER II.

5. *The Manometer, Anemometer, and Pitot Tube.*—To measure the work done by a fan we require two instruments to obtain the difference of pressure between suction and discharge, and the quantity of air passing through it per minute or per second; in this country it is usual to give the number of cubic feet per minute, but on the Continent

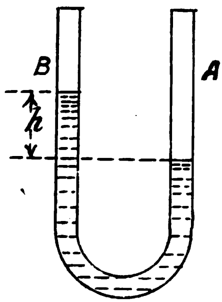


FIG. 3.

the number of cubic metres per second. A manometer is used to obtain the difference of pressure. If a bent tube, fig. 3, contains water and the end A is exposed to greater pressure than the end B, the liquid will rise on the latter side to a height proportional to the difference of pressure. At the average temperature a cubic foot of water weighs 62·3 lbs., so that each inch of gauge registers a pressure of

5·192 lb. per square foot. The gauge should not be placed in a current of air, as this will either increase or decrease the reading, because part of the equivalent head of the air is velocity head, and supposing the air at B is at rest, while that at A is in motion the height of the column at the side A will be greater than it should be by the amount

$$12 \frac{v^2}{2g} \frac{D}{C} \text{ inches,}$$

where D and C are the densities of the air and water; thus, if a fan is drawing air from a mine, and the manometer is in a strong current of air, A being open to the atmosphere, and B connected to the passage in which the air is approaching the fan, then h will be greater than it should be, and the fan will be credited with an amount of work that it has not done. On the other hand, suppose the fan is blowing air into the mine, and that B is open to the atmosphere, then the water gauge will be correct, because we cannot credit the fan with the energy of the air discharged, unless the discharge passage gradually increases, so as to reduce the velocity of the air before it reaches the mine, and if that is the case a tube should connect A to the end of the discharge passage. To take an extreme case, suppose a fan draws air direct from the atmosphere, and merely discharges it into the atmosphere again, then the fan is doing no useful work; but a manometer, whose end B was connected to the suction of the fan, and end A to the atmosphere where the air was at rest, would give a considerable reading, whereas the real water gauge should be zero, because the spaces from which the fan draws and into which it discharges are at the same pressure. The reading of the manometer is due to the reduction of pressure caused by the velocity of the air as it flows to the eye of the fan, and as the fan has the assistance of this amount of kinetic energy the gauge is incorrect. If the end of the manometer tube is ground square with its axis, and bent to face the current of air, the particles of air impinging upon the tube will raise the pressure by an amount

$$\frac{v^2}{2g} D \text{ per square foot,}$$

so that the error due to the reduction of pressure owing to the velocity of the air will be entirely neutralised, the water gauge being increased by the amount

$$\frac{12\,v^2}{2\,g}\,\frac{D}{C}\ \text{inches},$$

hence a manometer of this description may also be used to measure the quantity of air discharged by the fan, and is called a Pitot tube. The quantity of air is generally measured by an anemometer, fig. 4, which shows a type constructed by Messrs. Brady and Martin, of Newcastle, a small wheel carrying vanes set at an inclination to the plane of

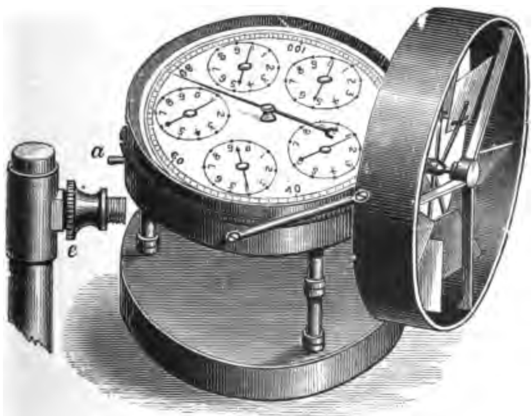


FIG. 4.

rotation so that when a current of air passes through the wheel in the direction of its axis the wheel will rotate, and communicate its motion to gearing which works a counter showing the number of revolutions. It was originally supposed that the number of revolutions was proportional to the number of turns of the wheel, but this is not the case even when the current of air has a uniform velocity, because of the friction of the apparatus, and is very far from being the case when it is variable; in the latter case the anemometer, which is usually graduated by rotating it in still air at the end of an arm driven at uniform velocity, greatly exaggerates the quantity of air passing through it. That this is the case will be readily seen by the reader when he considers the case of an anemometer which is alternately placed during equal short periods in a current of air, and in still air; when in the former it attains a speed very nearly proportional to it, and when in the latter it slows down very gradually, so that the total number of revolutions is considerably greater than would have been obtained had the instrument been placed in the current of air for half the time. So far, however, the anemometer is the best instrument invented for measuring the flow of air, and all that can be done is to place it where the velocity of flow is as uniform as possible.

We have already mentioned the Pitot tube; this is merely a manometer having one end of the tube ground square with the axis, which end is placed so that the axis is in the direction of the current, and the mouth of the tube is turned towards the current. The particles of air impinging on the mouth produce an additional pressure $\frac{v^2}{2g}$ D so that the water gauge is proportional to the square of the velocity, and therefore the velocity of the air can be measured, but the reader will see at once that if the velocity of the air varies, and this is usually the case, we shall get a water gauge which is proportional to the mean of the squares of the velocities, and not the square of the mean velocity, and thus the apparent discharge is greater than the real one.

6. *The Method of Calculating* H *from h, and Method of Calculating D.*—A water gauge of h inches corresponds to a pressure P per square foot where

$$P = \frac{C\,h}{12}\text{ lb.} \qquad \qquad (9)$$

and C = weight of one cubic foot of water at the temperature at which the experiment was made. If D is the weight of one cubic foot of air at that same temperature, to find which the height of the barometer, and the moisture contained by the air must be known, then the equivalont head of air against which the fan is working is

$$H = \frac{h\,C}{12\,D}\text{ feet} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and as C = 62·3 and D = ·075 at 62 Fah.

$$H = \frac{10{,}000}{144}\,h\text{ feet average} \quad . \quad (11)$$

where rough calculations only are necessary. Where greater accuracy is desired we require the barometer, thermometer, and hygrometer to obtain the correct value of D. When a mixture of two gases fills a space the pressure is the sum of the two pressures that would be produced by each of the gases filling the same space alone. A mixture of dry air and vapour has a pressure which is the sum of the pressures that they would produce if they filled the space alone. We can find the pressure of the vapour, because by the hygrometer we can find the dew point or temperature at which the amount of moisture in the air would just saturate it. To every temperature corresponds a certain vapour pressure, and these are tabulated, so that no formula is necessary; also corresponding to this pressure and temperature there is a certain density, so that we know from the table the weight of moisture per cubic foot. Let T be the pressure measured, in inches of mercury, of the moisture at the dew point, let its absolute temperature be θ^1, and let θ be the absolute temperature of the atmosphere in Fahrenheit degrees; let P be its pressure in inches of mercury, and σ the weight in pounds of a cubic foot of dry air at the standard barometric pressure of 29·92 ins. Then the weight of dry air in one cubic foot of the atmosphere is

$$M = \sigma\,.\,\frac{P - T}{29{\cdot}92}.$$

From the well known formula

$$p\,v = 53{\cdot}2\,\theta,$$

where p is in pounds per square feet, and v is the volume of 1 lb. in cubic feet, we can readily deduce that

$$\sigma = \frac{39 \cdot 8}{\theta}$$

whence

$$M = \frac{39 \cdot 8 \, (P - T)}{29 \cdot 92 \, \theta}.$$

This must be added to M¹, the weight of the moisture obtained from the table below, and the weight of one cubic foot of air or its density is

$$D = M + M^1.$$

If very great accuracy is desired account must be taken of the slight increase in length of the barometric column, due to the fact that the temperature θ at which it is read is not 32 deg. Fah., at which temperature only the standard barometer is 29·92. To allow for this we must write

$$M = \frac{39 \cdot 8 \, (P - T)}{29 \cdot 92 \, (1 + \cdot 0001 \, \{F - 32\}) \, \theta}.$$

Much assistance in writing the above has been obtained from Dr. Jude's Physics,* from which the following table has been copied :—

The following examples will make the method of obtaining the value of ρ clear :—

Example 1.—The temperature of the atmosphere is 77 deg. Fah., and the dew point as obtained by the hygrometer is 41 Fah., what is the weight of 1 cubic foot of air if the barometer is 29 in.? The pressure of the moisture is ·2572 in., so that

$$P - T = 28 \cdot 7428 \text{ and } M = \frac{\sigma \, (P - T)}{29 \cdot 92}$$

$$= \frac{39 \cdot 8 \times 28 \cdot 74}{538 \times 29 \cdot 92} = \cdot 071,$$

because $\theta = 77 + 461 = 538$ deg. absolute. Fah.

* "Physics: Experimental and Theoretical." By R. H. Jude, D.Sc., M.A. Chapman and Hall, publishers.

From the table we find that at 41 Fah. it requires 2,406 cubic feet of vapour to form 1 lb.

$$M^1 = \frac{1}{2406} = \cdot 00048 \text{ lb.}$$

so that the total weight of one cubic foot of air is

$$D = M + M^1 = \cdot 0714 \text{ lb.}$$

TABLE OF SATURATION PRESSURES AND VOLUMES OF AQUEOUS VAPOUR.

Temperature Fah.	Saturation pressure. Inches of mercury.	Saturation volume. No. of cub. ft. per lb.
32	·1811	3390
41	·2572	2406
50	·3608	1732
59	·5000	1264
68	·6846	935
77	·9279	699
86	1·2420	529
95	1·6470	405
104	2·1620	313
113	2·8110	244
122	3·6210	192
131	4·6260	152·4
140	5·8580	122
149	7·3580	98·45
158	9·1770	80·02
167	11·3600	65·47
176	13·9600	53·92

To obtain the absolute temperature θ add 461 to the Fahrenheit temperature.

Example 2.—The temperature of the air is 95 deg., and it is saturated with moisture, the barometer is at 29·92 in., what is the weight of 1 cubic foot of air? Here

$$P - T = 29{\cdot}92 - 1{\cdot}65 = 28{\cdot}27.$$

$$\theta = 95 + 461 = 556$$

$$M = \frac{39{\cdot}8\,(P - T)}{29{\cdot}92\,\theta} = \frac{39{\cdot}8 \times 28{\cdot}27}{29{\cdot}92 \times 556} = {\cdot}0676$$

$$M^1 = \frac{1}{405} = {\cdot}0025$$

$$D = M + M^1 = {\cdot}0701.$$

Example 3.—The temperature is 68 Fah., and the dew point is 50, the barometer is 29·7 in., what is the density of the air if we take into account the effect of temperature on the barometer?

$$P - T = 29{\cdot}7 - {\cdot}36 = 29{\cdot}34$$

$$\theta = 68 + 461 = 529,$$

$$M = \frac{39{\cdot}8\,(P - T)}{29{\cdot}92\,(1 + {\cdot}0001\,\{F - 32\})\,\theta} = \frac{39{\cdot}8 \times 29{\cdot}34}{29{\cdot}92 \times 529 \times 1{\cdot}0036}$$

$$= {\cdot}0737$$

$$M^1 = \frac{1}{1732} = {\cdot}0006 \text{ nearly}$$

$$D = {\cdot}0743 \text{ lb.}$$

CHAPTER III.

7. *The Law of Change of Moment of Momentum.*—One of the most important mechanical laws that applies to the fan is that the change of moment of the momentum of a mass acted upon by forces is equal to the moment of the impulse of the external forces, or to their angular impulse. If the weight of a body is W and its velocity is v its momentum is $\frac{W\,v}{g}$, and if r is the perpendicular from any point A, fig. 5, upon its direction of motion then $\frac{Wv}{g}\,r$

is the moment of its momentum, or its angular momentum. Suppose a force P to act upon the body during a very small time t; then by the second law of motion if V is the velocity produced in the direction of P

$$\frac{W\,V}{g} = P\,t,$$

and if r_1 is the perpendicular on P's direction then $P\,t\,r_1$ is the moment of the impulse of the force, or its angular impulse; and the change of the moment of momentum is $\frac{W\,V}{g}\,r_1$ because the moment of momentum of a body about a point is equal to the sum of the moments of the resolved parts of its momentum; therefore the change of the moment

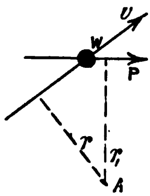


FIG. 5.

of the momentum is equal to the moment of the impulse of the external forces, because being true for a small period of time and a constant force, it is also true for a finite period and a variable force.

8. *On the Work done on the Air in its passage through a Radial Flow Fan.*—Imagine a mass of air passing through a wheel or fan, fig. 6, rotating about the axis C in the direction of the clock, and to simplify matters suppose that all particles follow similar paths such as A B and that the inflow at A is radial. Let c_1 be the velocity of the outer circumference in feet per second, and c_2 that of the inner, and let r_1, r_2 be the corresponding radii in feet. Let v be the absolute velocity of discharge and u_2 that of inflow just before the vanes act upon each particle, while u is the absolute velocity

just after; then the parallelograms of velocity of outflow and inflow are $v_1\, v\, c_1$ B and A $v_2\, u\, c_2$ so that v_2 and v_1 are the velocities of inflow and outflow, relative to the wheel. Then since the particle had no moment of momentum before it reached the wheel and since $\frac{W_1}{g}\, w_1\, r_1$ is its moment of momentum after leaving the wheel, therefore $\frac{W_1}{g}\, w_1\, r_1 =$

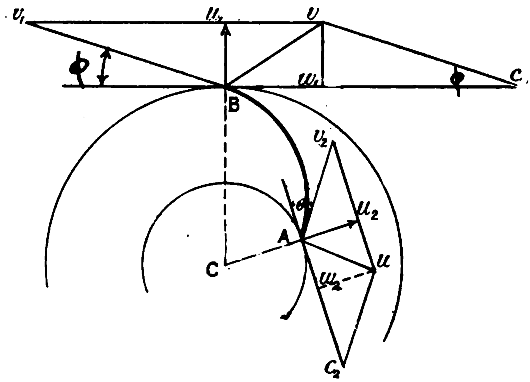


FIG. 6.

angular momentum of all forces acting on that particle, where W_1 is the weight of the particle. If then ω is the angular velocity of the wheel in radians and W is the total weight passing through the wheel per second,

$$\frac{W}{g}\, w_1\, r_1\, \omega = T\, \omega,$$

$$\frac{W}{g}\, w_1\, c_1 = T\, \omega,$$

= work per second transmitted to the wheel if T = total twisting moment in foot pounds.

Hence,

$$\frac{c_1\, w_1}{g} = \text{work done by wheel per pound of air} \quad . \quad (12)$$

and neglecting the friction of the bearings, which is never a very great quantity, this is the work done per pound of air on the fan shaft, however the air may approach the fan. For if no force acts on the air before it reaches the fan it can have no moment of momentum, and therefore must approach the fan radially, or if inflow is axial, axially, and hence the work done by the wheel, and that done on the shaft by the motor must be $\frac{c_1 w_1}{g}$, but if the friction of shaft or arms acts on the air and gives it angular momentum before reaching the wheel then $\frac{c_1 w_1}{g}$ is the work done by wheel vanes, arms, and friction of the shaft, and hence is that done by the motor on the shaft in addition to the small amount needed to overcome the bearing friction. Thus equation (12) not only applies to fans in which the flow through the wheel is wholly in a plane perpendicular to the axis of the shaft, or *radial flow fans*, but also to those in which the flow is changed from an axial direction to a radial direction or *mixed flow fans.*

9. *On the Losses of Energy or Head while Passing through the Fan.*—In passing through a fan there are several losses of head, which by proper design may either be entirely avoided or reduced to a minimum when the orifice or

$$\frac{Q}{\sqrt{gH}}$$

has the value for which the fan has been designed, where Q = cubic feet or metres per second, H = head of air in feet or metres given by formula (10) or (11), and g = 32·2 for British units, or 9·81 for metric units. Fig. 7 is an outline drawing of a fan showing two sectional elevations. In some fans there are two eyes A, in others one at which the air enters. It then passes through the wheel B, which rotates clockways, into the diffuser C. This diffuser is sometimes made with parallel sides, and often with a slight taper of about 7 deg. Its inner surface is always cylindrical, but its outer surface is sometimes of a spiral form, as in the Rateau ventilator to be described later. It is, however, usually cylindrical, but most fans are constructed without any diffuser at all, and the wheel B discharges directly into the volute D, which is usually of rectangular or circular section increasing from the beak E, according to a formula to be dealt with later, and having its greatest section at the base of the chimney F, which increases in section so as to reduce

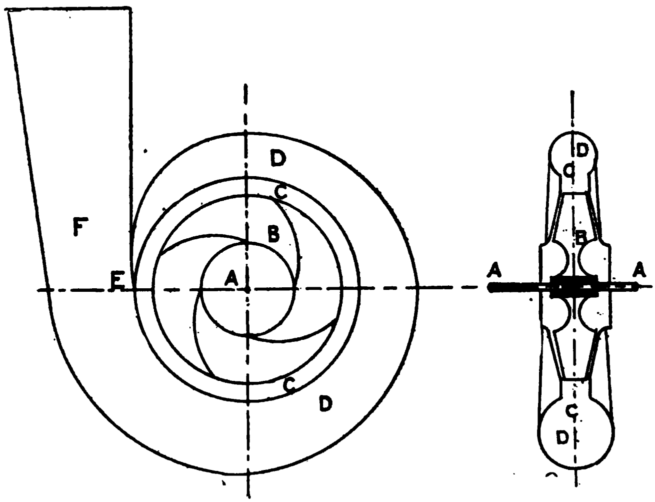


FIG. 7.

the velocity of discharge. Referring to fig. 6, we see that when the air enters the wheel at A its direction may be suddenly changed from u_2 to u, so that the loss of head is

$$L_1 = \frac{(u\, u_2)^2}{2\,g}$$

$$= \frac{(c_2 - u_2 \cot \theta)^2}{2\,g}$$

where $(u\, u_2)$ represents the length from u to u_2, and not the product of these two velocities, and θ is the angle made by the vane A B at A, with a tangent to the circle through A. In order that this loss of head may be avoided, we must make θ such that

$$\cot \theta = \frac{c_2}{u_2}$$

which we shall presently show, can only be the case for one value of

$$\frac{Q}{\sqrt{g\, H}}$$

After passing through the wheel the air enters either the atmosphere if the fan has no casing, or the diffuser, or if there is no diffuser, the volute ; in the first case the head lost is $\frac{v^2}{2\, g}$, as the kinetic energy at discharge is all lost; in the second case there is no loss when entering the diffuser, while in the third the loss is (fig. 8)

$$L_2 = \frac{(v\, v_4)^2}{2\, g}$$

$$= \frac{u_1^2 + (w_1 - v_4)^2}{2\, g} \quad . \quad . \quad (13)$$

where v_4 is the velocity in the volute which has a direction very nearly tangential to the wheel. If the fan has no chimney or expanding discharge pipe v_4 should $= \frac{1}{2}\, w_1$, but if it has then v_4 should $= w_1$ for the reasons given in paragraph 4, equations (4) and (6). If the fan has a diffuser, then if B D is its inner circumference, and C E its outer (fig. 9), the latter having a radius r_3, then the change of the angular momentum of each particle of air therein is nil, because no force acts on it during its passage through the diffuser, and if w_3, u_3 are the tangential and radial components at discharge, and b_1, b_3 the breadths of diffuser at inflow and discharge, then

$$w_1\, r_1 = w_3\, r_3$$

$$\frac{w_3}{w_1} = \frac{r_1}{r_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and $$2\pi r_1 b_1 u_1 = 2\pi r_3 b_3 u_3$$

$$\frac{u_3}{u_1} = \frac{b_1 r_1}{b_3 r_3} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and if the sides of the diffuser are parallel, or $b_1 = b_3$

$$\frac{u_3}{u_1} = \frac{r_1}{r_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

This, however, neglects the thicknesses of the vanes; the path B C is then an equi-angular spiral. But b_3 is usually made slightly greater than b_1, so that the sides are inclined to one another at an angle of about 7 deg.

The air next passes into the volute, and the loss of head is

$$L_3 = \frac{u_3^2 + (w_3 - v_4)^2}{2g} \quad . \quad . \quad . \quad . \quad (17)$$

and if there is an expanding discharge pipe we should have $v_4 = w_3$, and if not, $v_4 = \frac{1}{2} w_3$, according to paragraph 4, equations (4) and (6). In addition to the above there is the loss due to surface friction and bends which may be written

$$L_4 = F_1 \frac{v_1^2}{2g} + F_2 \frac{v_4^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where F_1, F_2 are constants depending on the proportions of the fan. There is also the loss of head due to the kinetic energy contained by the air at discharge.

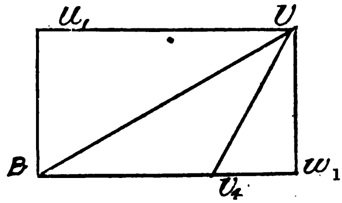


FIG. 8.

The values of v_4 in terms of w_1 and w_3 can only be obtained at one orifice, and make

$$L_2 = \frac{u_1^2}{2g}, \text{ or } \frac{u_1^2 + \frac{w_1^2}{4}}{2g} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and $$L_3 = \frac{u_3^2}{2g}, \text{ or } \frac{u_3^2 + \frac{w_3^2}{4}}{2g} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

10. *Equation for finding the Manometric and Mechanical Efficiencies in terms of ϕ.*—The work done by the wheel per second is equal to the head H multiplied by the weight of air per second, together with the work absorbed by losses of head; hence, if there is no casing

$$\frac{c_1 w_1}{g} - \text{losses of head} = H,$$

$$\frac{c_1 w_1}{g} - \frac{v^2}{2g} - F_1 \frac{v_1^2}{2g} - \frac{(c_2 - u_2 \cot \theta)^2}{2g} = H \quad . \quad . \quad (21)$$

$$w_1 = c_1 - u_1 \cot \phi,$$

ϕ being the angle between tangents to the curve A B at B, and the circle through B, and always measured clockways, fig. 6, and in some fans being greater than 90 deg., so that cot ϕ is negative.

$$v^2 = u_1^2 + w_1^2 = u_1^2 + (c_1 - u_1 \cot \phi)^2$$

$$v_1^2 = u_1^2 \operatorname{cosec}^2\phi = u_1^2 (1 + \cot^2\phi)$$

$$c_2 = c_1 \frac{r_2}{r_1},$$

so that it is evident that the equation can for a given fan be thrown into the form

$$c_1^2 + P c_1 Q - R Q^2 - S g H = o \quad . \quad . \quad . \quad . \quad (22)$$

where P, R, and S are constants containing F, ϕ, and θ, of which R and S are positive and P may be positive or negative. If the fan has a diffuser and no volute then

$$\frac{c_1 w_1}{g} - \frac{(c_2 - u_2 \cot \theta)^2}{2g} - \frac{F_1 v_1^2}{2g}$$

$$- \frac{u_2^2 + w_2^2}{2g}(1 + F_2) = H \quad . \quad . \quad . \quad . \quad (23)$$

If there is a volute and chimney but no diffuser

$$\frac{c_1 w_1}{g} - \frac{(c_2 - u_2 \cot \theta)^2}{2g} - \frac{F_1 v_1^2}{2g} - \frac{u_1^2}{2g} - \frac{(w_1 - v_4)^2}{2g} - \frac{F_2 v_4^2}{2g}$$

$$- \frac{v_5^2}{2g} = H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where v_5 is the velocity of discharge from the chimney.

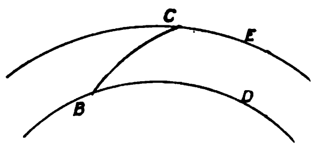


FIG. 9.

If there is a volute, chimney and diffuser

$$\frac{c_1 w_1}{g} - \frac{(c_2 - u_2 \cot \theta)^2}{2g} - F_1 \frac{v_1^2}{2g} - \frac{u_2^2}{2g} - \frac{(w_2 - v_4)^2}{2g} - F_2 \frac{v_4^2}{2g}$$

$$- \frac{v_5^2}{2g} = H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

and since all the velocities may be expressed in terms of c_1 and Q, the angles θ and ϕ, and the sections of the various passages of the fan we may transform equations (23), (24), (25) into the form given in (22). We must, however, mention that θ and ϕ are not exactly the angles of the vanes, but the angles of relative inflow and discharge, which from experiments made by the writer with the centrifugal pump, which only differs from the fan in that it pumps water instead of air, are greater than the vane angles the greater the discharge, and approach the vane angles as the discharge becomes small; the reader must remember that θ and ϕ are the angles made with the tangents to the inner and outer circumferences of the wheel, by the mean direction of flow to and from each passage (fig. 7). It is such points as these which make the differences, rarely very great, which are found between the results of experiment, and a theory, which is not absolutely exact, and which, to be so, would have to be expressed by extremely complicated equations.

We shall now consider the effect of varying ϕ in equation (21) to (25) on the mechanical efficiency of the fan, and on the manometric efficiency

$$M = \frac{g\,H}{c_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25a)$$

The mechanical efficiency does not differ much from the ratio of the useful work done per pound H to the work done by the wheel per pound $\frac{c_1\,w_1}{g}$. The difference is due to the friction of the wheel bearings, and to the friction of the outside casing of the wheel with the air which we have not included in the above equations, and which is only noticeable at very small orifices. We shall therefore find the ratio, which we shall call the air efficiency,

$$\eta = \frac{g\,H}{c_1\,w_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and we shall throughout take

$$u_1 = \cdot 5\sqrt{g\,H}\,. \quad . \quad . \quad . \quad . \quad (27)$$

when $\qquad c_2 = u_2 \cot\theta.$

Equation (21) may be thrown into the form

$$\frac{c_1\,(c_1 - u_1 \cot\phi)}{g} - \frac{u_1^2 + (c_1 - u_1 \cot\phi)^2}{2\,g} - F_1\,\frac{u_1^2 \operatorname{cosec}^2\phi}{2\,g} = H$$

$$c_1^2 - c_1\,u_1 \cot\phi - \tfrac{1}{2}\,u_1^2 - \tfrac{1}{2}\,c_1^2 + c_1\,u_1 \cot\phi - \tfrac{1}{2}\,u_1^2 \cot^2\phi$$
$$- \tfrac{1}{2}\,F_1\,u_1^2 \operatorname{cosec}^2\phi = g\,H$$

$$c_1^2 - u_1^2\,(1 + \cot^2\phi) - u_1^2\,F_1 \operatorname{cosec}^2\phi = 2\,g\,H$$

$$c_1^2 - (1 + F_1)\,u_1^2 \operatorname{cosec}^2\phi = 2\,g\,H$$

$$c_1^2 - \tfrac{1}{4}\,(1 + F_1)\,g\,H \operatorname{cosec}^2\phi = 2\,g\,H$$

$$M = \frac{1}{m^2} = \frac{g\,H}{c_1^2} = \frac{4}{8 + (1 + F_1)\operatorname{cosec}^2\phi} \quad . \quad . \quad (28)$$

and $$\eta = \frac{g\,H}{c_1(c_1 - u_1 \cot\phi)}$$

$$= \frac{1}{m(m - \frac{1}{2}\cot\phi)} \quad . \quad . \quad . \quad . \quad (29)$$

Considering next the case in which there is a diffuser but no volute, let r_3 the external radius of the diffuser be $k\,r_1$, and suppose the sides parallel to one another, then

$$\frac{c_1^2 - c_1 u_1 \cot\phi}{g} - F_1 \frac{u_1^2 \operatorname{cosec}^2\phi}{2g}$$

$$- \frac{u_1^2 + (c_1 - u_1\cot\phi)^2}{2g\,k^2}(1 + F_2) = H$$

$$2\,c_1^2 - 2\,c_1 u_1 \cot\phi - F_1 u_1^2 \operatorname{cosec}^2\phi$$

$$- \frac{1}{k^2}(1 + F_2)(u_1^2 + u_1^2\cot^2\phi + c_1^2 - 2\,c_1 u_1\cot\phi) = 2\,g\,H.$$

$$c_1^2\left[2 - \frac{(1 + F_2)}{k^2}\right] - 2\,c_1 u_1\cot\phi\left[1 - \frac{1}{k^2}(1 + F_2)\right]$$

$$- u_1^2 \operatorname{cosec}^2\phi\left[F_1 + \frac{1 + F_2}{k^2}\right] = 2\,g\,H.$$

and putting $u_1 = \frac{1}{2}\sqrt{g\,H}$,

$$c_1^2\left[2 - \frac{(1 + F_2)}{k^2}\right] - c_1\sqrt{g\,H}\cot\phi\left[1 - \frac{1}{k^2}(1 + F_2)\right]$$

$$- g\,H\left[2 + \frac{1}{4}\left(F_1 + \frac{1 + F_2}{k^2}\right)\operatorname{cosec}^2\phi\right] = o \quad . \quad (30)$$

If the fan has a volute and chimney, but no diffuser, and we suppose that $v_4 = w_1$ then equation (24) may be put in the form

$$2\,c_1^2 - 2\,c_1 u_1\cot\phi - F_1 u_1^2\operatorname{cosec}^2\phi - u_1^2 - F_2(c_1 - u_1\cot\phi)^2$$
$$- v_5^2 = 2\,g\,H.$$

Let $v_5 = \frac{1}{8}\sqrt{g\,H}$, then

$$c_1^2(2 - F_2) - c_1\sqrt{g\,H}\cot\phi\,(1 - F_2)$$

$$- g\,H\left(2 + \frac{1}{4} + \frac{1}{64} + \frac{F_1}{4}\operatorname{cosec}^2\phi + \frac{F_2}{4}\cot^2\phi\right) = o.$$

$$c_1^2(2 - F_2) - c_1 \sqrt{g H} \cot\phi (1 - F_2)$$
$$- g H \left(2\tfrac{7}{64} + \frac{F_1}{4} \operatorname{cosec}^2 \phi + \frac{F_2}{4} \cot^2 \phi\right) = o \quad . \quad (31)$$

If on the other hand $v_4 = \frac{1}{2} w_1$, and there is no chimney, then equation (24) becomes

$$2 c_1^2 - 2 c_1 u_1 \cot\phi - F_1 u_1^2 \operatorname{cosec}^2 \phi - u_1^2 - \tfrac{1}{4} (c_1 - u_1 \cot\phi)^2$$
$$- \tfrac{1}{4} F_2 (c_1 - u_1 \cot\phi)^2 - \tfrac{1}{4} (c_1 - u_1 \cot\phi)^2 = 2 g H$$

because v_5 is now the velocity of discharge from the volute, and therefore $\frac{1}{2} (c_1 - u_1 \cot\phi)$.

$$c_1^2\left(\tfrac{3}{2} - \frac{F_2}{4}\right) - c_1 u_1 \cot\phi \left(1 - \frac{F_2}{2}\right)$$
$$- u_1^2\left(1 + F_1 \operatorname{cosec}^2 \phi + \tfrac{1}{2} \cot^2\phi + \frac{F_2}{4} \cot^2 \phi\right) = 2 g H$$

and putting $u_1 = \cdot 5 \sqrt{g H}$

$$c_1^2\left(\tfrac{3}{2} - \frac{F_2}{4}\right) - \tfrac{1}{2}\left(1 - \frac{F_2}{2}\right) c_1 \sqrt{g H} \cot\phi$$
$$- \tfrac{1}{4} g H \left(9 + F_1 \operatorname{cosec}^2 \phi + \tfrac{1}{2} \cot^2\phi + \frac{F_2}{4} \cot^2 \phi\right) = o \quad (32)$$

If there is a diffuser, volute, and chimney, and if $r_3 = k r_1$, then

$$2 c_1^2 - 2 c_1 u_1 \cot\phi = F_1 u_1^2 \operatorname{cosec}^2 \phi$$
$$- \frac{u_1^2}{k^2} - F_2 \frac{(c_1 - u_1 \cot\phi)^2}{k^2} - \frac{g H}{64} = 2 g H$$

recollecting that
$$v_4 = w_3 = \frac{w_1}{k}$$

$$c_1^2\left(2 - \frac{F_2}{k^2}\right) - 2 c_1 u_1 \cot\phi \left(1 - \frac{F_2}{k^2}\right)$$
$$- u_1^2\left(\frac{1}{k^2} + F_1 \operatorname{cosec}^2 \phi + F_2 \frac{\cot^2\phi}{k^2}\right) - \frac{g H}{64} - 2 g H = o$$

$$c_1^2\left(2 - \frac{F_2}{k^2}\right) - c_1 \sqrt{g H} \cot\phi \left(1 - \frac{F_2}{k^2}\right)$$
$$- g H \left(\frac{1}{4 k^2} + \frac{F_1 \operatorname{cosec}^2 \phi}{4} + \frac{F_2 \cot^2 \phi}{4 k^2} + 2\tfrac{1}{64}\right) = 0 \quad . \quad (33)$$

The values of F_1, F_2 that agree best with practice are both $\frac{1}{8}$. Substituting this in (28), we obtain

$$c_1^2 - \cdot 281\, g\,\text{H}\,\text{cosec}^2\,\phi = 2\, g\,\text{H} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

from which the following table is calculated :—

ϕ = 15	$c_1 = 2{\cdot}47\,\sqrt{g\,\text{H}}$	M = ·164	η = ·66
= 30	1·76	·32	·63
= 45	1·60	·39	·57
= 90	1·51	·41	·41

so that the efficiency of an open-running fan without diffuser or volute is very low unless the vanes are curved backwards, and it must be remembered that to get the mechanical efficiency of engine and fan we must multiply η by ·85 on the average, so that the greatest mechanical efficiency of the combination is ·56 per cent. Taking next the case of the open-running fan with a diffuser whose sides are parallel and whose external radius is $1\frac{1}{4}$ that of the fan, that is

$$r_3 = 1\tfrac{1}{4}\, r_1$$

equation (30) becomes

$$1{\cdot}28\, c_1^2 - \cdot 28\, c_1 \sqrt{g\,\text{H}}\,\cot\phi - g\,\text{H}\,(2{\cdot}03 + \cdot 18\,\text{cosec}^2\,\phi) = o \quad . \quad (35)$$

which gives the following table :—

ϕ = 15	$c_1 = 2{\cdot}36\,\sqrt{g\,\text{H}}$	M = ·18	η = ·84
30	1·66	·36	·75
45	1·48	·45	·68
90	1·31	·58	·58

so that the highest possible mechanical efficiency of engine and fan would be 71 per cent. It must clearly be understood that ϕ is not the vane angle, but the angle of flow, and this will probably be from 15 deg. to 30 deg. greater than the vane angle. When the fan has no diffuser but a volute and chimney, and the former of such a section that $v_4 = w_1$, then (31) becomes

$$1{\cdot}875\, c_1^2 - \cdot 875\,\cot\phi\, c_1 \sqrt{g\,\text{H}} - g\,\text{H}\left(2{\cdot}297 + \frac{\cot^2\phi}{16}\right) = o \quad . \quad (36)$$

from which we obtain

$\phi = 15$	$c_1 = 2{\cdot}43\sqrt{g\mathrm{H}}$	$\mathrm{M} = {\cdot}168$	$\eta = {\cdot}72$
30	1·61	·38	·83
45	1·37	·53	·83
90	1·10	·81	·81
120	·99	1·02	·79
135	·91	1·20	·78

giving a maximum efficiency of engine and fan of 70 per cent.

If, however, there is no chimney, but the air is discharged direct from the volute, and $v_4 = \frac{1}{2}w_1$, then (32) becomes

$$1{\cdot}47\,c_1^2 - \frac{15}{32}c_1\sqrt{g\mathrm{H}}\cot\phi - \frac{g\mathrm{H}}{4}\left(9\tfrac{1}{8} + \frac{21\cot^2\phi}{32}\right) = o \quad . \quad (37)$$

which gives us the following table :—

$\phi = 15$	$c_1 = 2{\cdot}45\sqrt{g\mathrm{H}}$	$\mathrm{M} = {\cdot}166$	$\eta = {\cdot}69$
30	1·67	·36	·74
45	1·46	·48	·70
90	1·24	·64	·64

giving a maximum efficiency of engine and fan of 63 per cent.

Finally, if there are diffuser, volute, and chimney,

$$1{\cdot}92\,c_1^2 - {\cdot}92\cot\phi\,c_1\sqrt{g\mathrm{H}} - g\mathrm{H}\,(2{\cdot}2 + {\cdot}051\cot^2\phi) = o \quad . \quad (38)$$

and we get the following table :—

$\phi = 15^\circ$	$c_1 = 2{\cdot}41\sqrt{g\mathrm{H}}$	$\mathrm{M} = {\cdot}172$	$\eta = {\cdot}75$
30	1·59	·39	·86
45	1·35	·55	·87
60	1·22	·67	·87
90	1·07	·87	·87
120	·94	1·13	·87
135	·87	1·33	·84

giving a maximum efficiency of engine and fan of 74 per cent.

A fan designed in this manner would require a very long chimney, as the taper of a chimney cannot be very great, in order to reduce the velocity of the air to so low a value as $\frac{1}{8}\sqrt{g\mathrm{H}}$. We shall therefore consider the case in which the external radius of the diffuser is $1\frac{1}{2}$ that of the wheel, and the velocity v_4 in the volute is $\frac{1}{2}w_2$. Equation (25) then becomes, when we put

$$c_2 = u_2 \cot \theta, \qquad w_2 = \frac{c_1 - u_1 \cot \phi}{1\cdot 5},$$

and $u_1 = \cdot 5 \sqrt{g\,H},$

$$1\cdot 875\, c_1^2 - \tfrac{7}{8}\, c_1 \sqrt{g\,H} \cot \phi - g\,H\,(2\cdot 2 + \cdot 0625 \cot^2 \phi) = o \;. \;(38a)$$

from which we obtain the following table :—

$\phi =$ 15°	$c_1 = 2\cdot 41 \sqrt{g\,H}$	M = ·172	$\eta =$ ·75
= 30	1·60	·391	·85
= 45	1·35	·55	·872
= 90	1·08	·86	·86
= 120	·96	1·085	·835
= 135	·885	1·275	·815

CHAPTER IV.

11. *Theoretical Characteristics of Fans.*—If M is the manometric efficiency $\frac{g\,H}{c_1^2}$, and $\frac{Q}{\sqrt{g\,H}} = O$ the orifice, Q being cubic feet or cubic metres, and H feet or metres, then

$$c_1^2 + P\,c_1\,Q - R\,Q^2 - S\,g\,H = o \quad . \; . \; . \quad (22)$$

it may be put in the form

$$\frac{1}{M} + \frac{P\,O}{\sqrt{M}} - R\,.\,O^2 - S = o \; . \; . \; . \quad (39)$$

which is a quadratic for $\frac{1}{\sqrt{M}}$, whose solution is

$$\frac{1}{\sqrt{M}} = \frac{\sqrt{O^2\,(P^2 + 4\,R) + 4\,S} - O\,P}{2} \quad . \; . \quad (40)$$

If a curve is drawn with O as abscissa and M as ordinate, and another with O as abscissa and the mechanical efficiency of the fan alone as ordinate, we get two characteristic curves, which are very useful in showing the excellence or otherwise of the design.

We shall now consider the characteristic curves of a fan with no diffuser, but a volute and chimney having $\phi = 30$ deg. Assume that the external radius r_1 is three times the internal r_2, and that the velocity of inflow u_2 is radial, and equal to $\cdot 5 \sqrt{g\,\mathrm{H}}$ when

$$c_2 = u_2 \cot \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$
$$= \tfrac{1}{3} c_1 = \tfrac{1}{3} \times 1{\cdot}61 \sqrt{g\,\mathrm{H}}$$
$$\therefore \cot \theta = 1{\cdot}07.$$

Also at this orifice $v_4 = w_1 = c_1 - u_1 \cot \phi = (1{\cdot}61 - {\cdot}5 \times 1{\cdot}73) \sqrt{g\,\mathrm{H}} = {\cdot}75 \sqrt{g\,\mathrm{H}}$, so that at any discharge we shall always have

$$v_4 = 1{\cdot}5\, u_1,$$

because if a_1, a_2, etc., are the sections of passages in a machine through which a liquid flows, and V_1, V_2, etc., the corresponding mean velocities of flow, then

$$a_1 \mathrm{V}_1 = a_2 \mathrm{V}_2 = a_3 \mathrm{V}_3 = \quad . \quad . \quad . \quad . \quad (41)$$

Again, if $v_5 = \tfrac{1}{8} \sqrt{g\,\mathrm{H}}$ when $u_1 = \tfrac{1}{2} \sqrt{g\,\mathrm{H}}$,

$$v_5 = \tfrac{1}{4} u_1,$$

so that (24) becomes

$$2\,c_1^2 - 2\,c_1 u_1 \cot \phi - \frac{c_1^2}{9} + \tfrac{2}{3} c_1 u_1 \cot \theta - u_1^2 \cot^2 \theta$$
$$- \tfrac{1}{8} u_1^2 \operatorname{cosec}^2 \phi - u_1^2 - (c_1 - u_1 \cot \phi - 1{\cdot}5\, u_1)^2$$
$$- {\cdot}28\, u_1^2 - {\cdot}0625\, u_1^2 = 2\,g\,\mathrm{H}.$$

$$\tfrac{8}{9} c_1^2 + (3 + \tfrac{2}{3} \cot \theta)\, c_1 u_1$$
$$- u_1^2 (\cot^2 \theta + 1\tfrac{1}{8} \cot^2 \phi + 3 \cot \phi + 3{\cdot}7175) = 2\,g\,\mathrm{H} \quad . \quad (42)$$

$$\frac{1}{\mathrm{M}} + \tfrac{9}{8} (3 + \tfrac{2}{3} \cot \theta) \frac{1}{\sqrt{\mathrm{M}}} \frac{u_1}{\sqrt{g\,\mathrm{H}}}$$
$$- \tfrac{9}{8} (\cot^2 \theta + 1\tfrac{1}{8} \cot^2 \phi + 3 \cot \phi + 3{\cdot}7175) \frac{u_1^2}{g\,\mathrm{H}} - \tfrac{9}{4} = o.$$

Putting $\phi = 30$, $\cot\theta = 1{\cdot}07$, this becomes

$$\frac{1}{M} + 4{\cdot}177 \frac{1}{\sqrt{M}} \frac{u_1}{\sqrt{g\,H}} - 15{\cdot}1 \frac{u_1^2}{g\,H} - \tfrac{9}{4} = o.$$

$$\frac{1}{\sqrt{M}} = \frac{\sqrt{(17{\cdot}45 + 60{\cdot}4)\frac{u_1^2}{g\,H} + 9} - 4{\cdot}177 \frac{u_1}{\sqrt{g\,H}}}{2}$$

$$= \frac{\sqrt{(77{\cdot}85 \frac{u_1^2}{g\,H} + 9)} - 4{\cdot}177 \frac{u_1}{\sqrt{g\,H}}}{2} = \frac{c_1}{\sqrt{g\,H}} \quad . \quad (43)$$

From the above we can find w_1, and thence η, so that the following table is readily obtained; only it must be remembered that for small orifices the fan efficiency is really less than $\frac{g\,H}{c_1\,w_1}$, owing to external friction between the outer surface of the wheel casing and the air, or, if the wheel is open at the sides, between the fan casing and the air, which friction $\propto c_1^2$ in any fan, and the work wasted $\propto c_1^3$. If the efficiency were equal to $\frac{g\,H}{c_1\,w_1}$, then at zero orifice it would be equal to M, whereas its real value is zero.

$\frac{u_1}{\sqrt{g\,H}}$	$\frac{c_1}{\sqrt{g\,H}}$	M	η
0	1·5	·44	0
·1	1·345	·548	·63
·2	1·325	·562	·77
·3	1·375	·528	·847
·4	1·43	·49	·94
·5	1·62	·372	·82
·6	1·78	·314	·74
·8	2·16	·211	·59
1·0	2·56	·152	·47
1·5	3·65	·078	·25
2·0	4·80	·043	·16

If $\phi = 90$ deg., then, when there is no shock at inflow, $c_1 = 1 \cdot 1 \sqrt{g \text{H}}$, and therefore

$$\cot \theta = \frac{c_2}{u_2} = \frac{1 \cdot 1}{1 \cdot 5} = \cdot 73 \, ;$$

also at this orifice $v_4 = w_1 = c_1 = 1 \cdot 1 \sqrt{g \text{H}}$;

$$\therefore v_4 = \frac{1 \cdot 1}{\cdot 5} u_1 = 2 \cdot 2 \, u_1 \text{ at all orifices,}$$

and $$v_5 = \frac{u_1}{4} ,$$

so that (24) becomes

$$c_1^2 + 5 \cdot 49 \, c_1 u_1 - 8 \cdot 06 \, u_1^2 - \frac{9}{4} g \text{H} = o.$$

$$\frac{c_1}{\sqrt{g \text{H}}} = \frac{\sqrt{62 \cdot 38 \frac{u_1^2}{g \text{H}} + 9} - 5 \cdot 49 \frac{u_1}{\sqrt{g \text{H}}}}{2} \quad . \quad (44)$$

from which we obtain the following table :—

$\frac{u_1}{\sqrt{g \text{H}}}$	$\frac{c_1}{\sqrt{g \text{H}}}$	M	η
0	1·5	·44	0
·1	1·275	·608	·608
·2	1·150	·756	·756
·3	1·08	·846	·846
·4	1·08	·846	·846
·5	1·10	·810	·810
·6	1·15	·757	·757
·8	1·30	·598	·539
1·0	1·47	·449	·449
1·5	1·99	·250	·250
2·0	2·54	·152	·152

If $\phi = 135$, then, when there is no shock at inflow, $c_1 = \cdot 91 \sqrt{g\,H}$, and therefore

$$\cot \theta = \frac{c_2}{u_2} = \frac{\cdot 91}{1\cdot 5} = \cdot 6\,;$$

also at this orifice

$$v_4 = w_1 = c_1 - u_1 \cot \phi = c_1 + v_1 = 1\cdot 41 \sqrt{g\,H},$$

so that $v_4 = 2\cdot 82\, u_1$ at all orifices, and $v_5 = \frac{u_1}{4}$, so that (24) becomes

$$c_1^2 + 6\cdot 79\, c_1\, u_1 - 6\cdot 71\, u_1^2 - \frac{9}{4} g\,H = o\,;$$

$$\therefore \frac{c_1}{\sqrt{g\,H}} = \frac{\sqrt{72\cdot 94 \frac{u_1^2}{g\,H} + 9} - 6\cdot 79 \frac{u_1}{\sqrt{g\,H}}}{2} \quad . \quad (45)$$

from which the following table is calculated :—

$\frac{u_1}{\sqrt{g\,H}}$	$\frac{c_1}{\sqrt{g\,H}}$	M	η
0	1·5	·44	0
·2	1·045	·90	·77
·4	·91	1·21	·83
·5	·91	1·2	·78
·8	1·015	·99	·54
1·0	1·130	·77	·41
2·0	1·88	·53	·137

Hence in this type of fan there is a considerable advantage in making the vanes curve forward at discharge—that is, in making ϕ equal to or greater than 90 deg.—for a water gauge of 1 in. corresponds to a head of air of about 70 ft., and therefore a high manometric efficiency is desirable if it does not sacrifice the mechanical efficiency to any serious extent, because it allows of the fan being driven direct from the engine, and thus avoids the friction of belting or ropes. It also frequently happens that the orifice at which a fan has to work, during the time it is in use, is not constant; for example, owing to the enlargement of a mine, more air may be required at the same gauge. Now, with vanes having ϕ greater than 90 deg., the manometric efficiency between the orifice at which $u_1 = \cdot 4 \sqrt{g \text{H}}$ and that at which $u_1 = \cdot 8 \sqrt{g \text{H}}$ is much more nearly constant than when $\phi = 30$ deg. This type of fan must have an efficient chimney, in which the gain of pressure head is equal to the loss of velocity head. If the flow of air to a chimney is not uniform, or the inclination of its sides is too great, its efficiency will be very low. We shall give later some experiments made by Messrs. Heenan and Gilbert on the efficiency of chimneys.

If the fan has a diffuser, its efficiency at a given orifice will obviously be practically the same when $\phi = 135$ deg. as when $\phi = 30$ deg., because the principal cause of difference is the loss of head in the volute, due to friction, and, owing to the diffuser, the velocity is small and the loss of head equally so. Where there is not space to fit a good chimney (*e.g.*, on board ship), then a fan must be used in which $v_4 = \frac{1}{2} w_1$, and $\phi = 30$ deg., so that a high efficiency may be obtained at a constant orifice; the vane angle at discharge might then be about 15 deg.

The reader may possibly imagine that the above tables might be altered by a change in the ratio $\frac{u_2}{u_1}$. This, however, is not the case, because $u_2 \cot \theta = c_2$ when $u_1 = \cdot 5 \sqrt{g \text{H}}$; hence the term $u_2 \cot \theta$ will be unchanged whatever $\frac{u_2}{u_1}$ may be. In the above we have assumed that the external radius r_1 of the wheel is three times the internal r_2, and a change in this will slightly affect the coefficients of $c_1 u_1$, u_1^2, and g H. But whatever the ratio $\frac{r_1}{r_2}$ may be, the real manometric efficiency at zero orifice should not differ much from $\frac{1}{2}$, because the air within the eye is really rotating with the same angular velocity as that within the fan, and under these circumstances $c_1 = \sqrt{2 g \text{H}}$. When $u_1 = \cdot 5 \sqrt{g \text{H}}$

the mechanical and manometric efficiencies are entirely independent of $\frac{r_1}{r_2}$, because $c_2 - u_2 \cot \theta = o$.

Volumetric Efficiency.—If a fan were made very large in proportion to its discharge, it would no doubt give a high mechanical efficiency, owing to the fact that the loss due to the friction of air in the passages would be extremely small; but the cost of such a fan would probably more than outweigh the advantage thus obtained. It is, therefore, necessary for designers to have another ratio, which is usually called the volumetric efficiency, which compares the discharge with the tip speed c_1 and the dimensions of the fan. Let Q be the discharge in cubic feet or metres, c_1 and r_1 the tip speed and external radius in feet or metres, then, if the fan has one eye, the volumetric efficiency is

$$\delta = \frac{Q}{c_1 r_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

and if it has two eyes—*i.e.*, one on each side—

$$\delta = \frac{Q}{2 c_1 r_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46a)$$

which is undoubtedly correct for propeller and screw, but is not so for radial and mixed flow fans. In the two former the space occupied by the wheel, and through which the air has to pass, is πr_1^2, but in the latter the air passage is not. For example, a fan having a small diameter and moderate breadth at outflow may have a higher volumetric efficiency than one which has a large diameter and small breadth, although the circumferential area through which the air flows may be the same in both, and they may be so designed that $\frac{Q}{c_1 r_1^2}$ may be the same for both. For this reason we consider that a better measure of the volumetric efficiency is

$$\delta_1 = \frac{Q}{c_1 r_1 b_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where b_1 is the breadth of the wheel internally at the external radius.

In the following pages we shall mean by volumetric efficiency the quantity δ, and shall denote the more correct value by the letter δ_1. These efficiencies can be greater than unity.

CHAPTER V.

13. *Design of Radial Flow Fans.*—Fans may be divided into three classes, viz., those in which the flow through the wheel is in a plane perpendicular to the axis, or radial flow fans; those in which the inflow is axial and the direction is gradually changed in the wheel to a radial one, these being named mixed flow fans; and those in which the flow is axial, which are usually spoken of as screw or propeller fans. We shall now consider the design of the first of these three.

We shall first consider the value $\frac{u_1}{\sqrt{g\,H}}$.

Messrs. Heenan and Gilbert† found that with a fan 17 in. diameter and 8 in. broad the discharge was about 3,400 cubic feet of air per minute at the most suitable orifice; the water gauge was 9·3 in., and consequently

$$u_1 = \frac{Q}{2\,\pi\,r_1\,b_1} = \frac{3400 \times 144}{\pi \times 17 \times 8 \times 60},$$

$$g\,H = \frac{32 \times 9{\cdot}3 \times 10000}{144}$$

at average temperature and pressure; hence

$$\frac{u_1}{\sqrt{g\,H}} = {\cdot}134 \text{ very nearly.}$$

In Mr. Bryan Donkin's paper on Centrifugal Fans (Proc. Inst. of C.E., vol. cxxii., fan No. 1),‡ a Rateau fan with an

† See fig. 8, plate 5 of a paper on Centrifugal Fans, by Messrs. Heenan and Gilbert, in the Proceedings of the Institution of Civil Engineers, vol. cxxii.

‡ *Book publication only.* See fig. 41 further on.

external radius of 9·8 in., has its maximum efficiency at an equivalent orifice of ·4 square feet. The equivalent orifice

$$O_1 = \frac{Q}{\cdot 65\sqrt{2\,g\,H}} = 1{\cdot}088\,\frac{Q}{\sqrt{g\,H}} \quad . \quad (48)$$

·65 being an assumed coefficient of contraction. In the Rateau ventilator

$$2\,\pi\,r_1\,b_1 = r_1^2$$

so that

$$\frac{u_1\,r_1^2}{\sqrt{g\,H}} = \frac{Q}{\sqrt{g\,H}}$$

$$\frac{u_1}{\sqrt{g\,H}} = \frac{\cdot 4 \times 144}{1{\cdot}088 \times (9{\cdot}8)^2} = \cdot 55$$

so that $\frac{u_1}{\sqrt{g\,H}}$ varies in practice between considerable limits, and we may assume it to be what we please in our design. Again, the velocity at the eye of the Rateau fan is also ·55 $\sqrt{g\,H}$. In the Heenan fan inflow takes place at both sides, and the diameter of the eye is 9 in.; the velocity of inflow is therefore

$$\frac{\cdot 134 \times \pi \times 8 \times 17}{2 \times \frac{\pi}{4} \times 9^2}\sqrt{g\,H} = \cdot 45\sqrt{g\,H}.$$

Again, the radial component of the velocity of inflow at the inner radius is

$$\frac{\cdot 134 \times \pi \times 8 \times 17}{\pi \times 8 \times 9}\sqrt{g\,H} = \cdot 253\sqrt{g\,H}.$$

Let us first consider the design of an open-running fan which is required to deliver 60,000 cubic feet of air per minute with a water gauge of 2 in. Here

$$\frac{Q}{\sqrt{g\,H}} = \frac{1000}{8 \times \frac{100}{12}} = \frac{120}{8} = 15.$$

Assume that the velocity of inflow at the sides is

$$\cdot 5\sqrt{g\,H} = \frac{400}{12} = 33{\cdot}33 \text{ ft. per sec.}$$

Let r_4 be the radius of the eye, then

$$2 \pi r_4^2 \times 33{\cdot}33 = 1000 = Q$$

$$r_4^2 = \frac{500}{33{\cdot}33 \pi}$$

$$r_4 = 2{\cdot}18 \text{ ft.}$$

The internal radius of the vanes should be made slightly larger than this, say $2\frac{1}{2}$ ft. Let the velocity of inflow be $\cdot 5 \sqrt{g H}$. The thickness of the vanes may be neglected, and we have

$$2 \pi r_2 b_2 \times \cdot 5 \sqrt{g H} = 1000 ;$$

where b_2 is the effective breadth of the wheel at radius r_2,

$$b_2 = 1{\cdot}91 \text{ ft.}$$

If the radial component at outflow $u_1 = \cdot 5 \sqrt{g H}$, and the vanes are made to touch the circumference at the outer radius, then we have already shown that $\eta = \cdot 66$ and $c_1 = 2{\cdot}47 \sqrt{g H} = 164{\cdot}66$ ft. per second. If the revolutions N per minute are fixed, we can calculate r_1 from the formula

$$2 \pi r_1 N = 60 c_1,$$

but if not, let $r_1 = 2 r_2 = 5$ ft.; then

$$N = \frac{60 \times 164{\cdot}66}{10 \pi} = 314 \text{ revs. per minute,}$$

and the effective breadth at the outer radius

$$b_1 = b_2 \frac{r_2}{r_1} = \cdot 955 \text{ ft.}$$

Of course all these dimensions must be reduced to their nearest values in feet and inches.

If u_1 does not $= \cdot 5 \sqrt{g H}$, we shall have another value of c, which must be deduced from equation (21); since

$$c_2 = c_1 \frac{r_2}{r_1} = 1{\cdot}235 \sqrt{g H},$$

$$\cot \theta = \frac{c_2}{u_2} = \frac{1{\cdot}235}{\cdot 5} = 2{\cdot}47.$$

The brake horse power required to drive the fan is

$$\text{B.H.P.} = \frac{62{\cdot}3\ Q\ h}{12 \times 550 \times \eta} = \frac{62300 \times 2}{12 \times 550 \times {\cdot}66} = 28{\cdot}6,$$

so that the indicated horse power of the engine is

$$\text{I.H.P.} = \frac{\text{B.H.P.}}{{\cdot}85} = 33{\cdot}6.$$

Next suppose our fan is provided with a diffuser; the vane should be a tangent to the outer circumference, so that, allowing for the divergence of the air, $\phi = 15$ deg., $c_1 = 2{\cdot}36\ \sqrt{g\,H}$, $N = 300$, and $\cot\theta = 2{\cdot}36$.

The diffuser may be made with its sides diverging from one another about 7 deg., corresponding to an inclination to the vertical of about $\frac{1}{16}$, and its external radius is

$$r_3 = 1\tfrac{1}{2}\ r_1 = 6\tfrac{1}{4}\text{ ft.}$$

The diffuser may either rotate with the wheel or form part of the casing, and the sides of the wheel should not form a simple frustrum of a cone, but a radial section should be concave when looked at from the outside, so that the air may have no motion parallel to the axis when discharged from the wheel into the diffuser. The breadth of the diffuser at the external radius is

$$b_3 = b_1 + \tfrac{1}{8}\,(r_3 - r_1) = 1{\cdot}11\text{ ft.}$$

The brake horse power is now

$$\text{B.H.P.} = \frac{62{\cdot}3 \times 1000 \times 2}{12 \times 550 \times {\cdot}85} = \frac{18{\cdot}88}{{\cdot}85} = 22{\cdot}2,$$

and

$$\text{I.H.P.} = \frac{\text{B.H.P.}}{{\cdot}85} = 27{\cdot}3.$$

If the fan has no diffuser, or chimney, but a volute in which $v_4 = \frac{1}{2}\,w_1$, then, as the highest efficiency is obtained with $\phi = 30$ deg., the vane angle at discharge should be 15 deg., $c_1 = 1{\cdot}67\ \sqrt{g\,H}$, $N = 212$ revolutions per minute, and $\cot\theta = 1{\cdot}67$, $\text{B.H.P.} = \frac{18{\cdot}88}{{\cdot}74} = 25{\cdot}51$, $\text{I.H.P.} = 30$, $w_1 = c_1 - u_1 \cot\phi = {\cdot}81\ \sqrt{g\,H} = 54$. The section of the volute is proportional to the angular distance from its beak E, fig. 7, because the discharge from the fan is uniform all round the circumference, and if ψ is this angular distance, s the section, and S is its section at discharge, then

$$s = \frac{\psi}{2\pi} S$$

and
$$S = \frac{Q}{v_4} = \frac{2Q}{w_1} = \frac{2000}{54} = 37 \text{ square feet},$$

corresponding to a diameter of 6·87 ft. The section, however, need not be circular; a rectangular form is frequently used. In some cases the wheel discharges into the centre of this section; in others, the volute is wholly outside the wheel. If the fan has a chimney, then $w_1 = v_4$, but in this case it is possible to obtain a high manometric efficiency. We shall consider the design in two cases: firstly, when $\phi = 90$ deg.; secondly, when it is 120 deg. In the former case, we believe that it is necessary to make the vane angle about 105 deg., so that the average flow may be radial, if the number of vanes are few, say six; but if twelve vanes are used, then the vanes may be radial at discharge. In either case, we may assume $c_1 = w_1 = 1 \cdot 1 \sqrt{g H}$, N = 139 revolutions per minute. The section of the volute at discharge is

$$S = \frac{Q}{w_1} = \frac{1000}{73 \cdot 33} = 13 \cdot 63 \text{ square feet},$$

while the larger en l of the chimney has a section

$$S_1 = \frac{Q}{v_5} = \frac{1000}{\frac{1}{8}\sqrt{g H}} = 120 \text{ square feet.}$$

$$\text{B.H.P.} = \frac{18 \cdot 88}{\cdot 81} = 23 \cdot 3$$

and
$$\text{I.H.P.} = \frac{\text{B.H.P.}}{\cdot 85} = 27 \cdot 4$$

$$\cot \theta = 1 \cdot 1.$$

If $\phi = 120$ deg. the vane angle should be 135 deg. if there are six vanes, and 120 deg. if there are twelve;

then $c_1 = \cdot 99 \sqrt{g\,H}$, N = 126, and $\cot \theta = \cdot 99$, B.H.P. = $\frac{18\cdot 88}{\cdot 79} = 23\cdot 9$, I.H.P. = $\frac{\text{B.H.P.}}{\cdot 85} = 28\cdot 1$, $w_1 = 1\cdot 278 \sqrt{g\,H}$. $S = \frac{Q}{w_1} = \frac{1000}{85\cdot 2} = 11\cdot 7$ square feet. $S_1 = 120$ square feet as before.

We now come to the case where there is a diffuser, volute, and chimney, and shall assume $\phi = 120$ deg., making the vane angle 135 deg. if there are six vanes, and 120 deg. if there are twelve. The external radius r_3 of the volute is to be $1\frac{1}{4}$ that of the wheel, and therefore

$$r_3 = 6\tfrac{1}{4} \text{ ft.}$$

$c_1 = \cdot 94 \sqrt{g\,H}$, N = 119, $\cot \theta = \cdot 94$, B.H.P. = $\frac{18\cdot 88}{\cdot 87} = 21\cdot 7$, I.H.P. = $\frac{21\cdot 7}{\cdot 85} = 25\cdot 5$. The breadth of the diffuser is as before 1·11 ft., and since

$$w_1 = c_1 - u_1 \cot \phi = (\cdot 94 + \cdot 288) \sqrt{g\,H} = 1\cdot 228 \sqrt{g\,H}.$$

$$w_3 = \frac{r_1}{r_3} w_1 = \tfrac{4}{5} \times 1\cdot 228 \sqrt{g\,H} = 68\cdot 6.$$

Hence $S = \frac{1000}{68\cdot 6} = 14\cdot 5$ square feet, and the large end of the chimney 120 square feet as before.

It is the practice of some fan makers to make the sides of the wheel parallel, because no doubt it is very much cheaper; there must be, however, a considerable loss of head at inflow, and eddies must be formed, because the velocity of flow at the centre of the wheel must be greater than that at the sides. On the other hand, there is less loss of head at outflow from the wheel owing to $\frac{u_1^2}{2g}$ being very much less than $\frac{u_2^2}{2g}$. Anyone who has read the above designs will have no difficulty in calculating the dimensions of such a fan; but to be strictly accurate he must not assume any of the tabulated values of $\frac{c_1}{\sqrt{g\,H}}$, but must find them from equations (21) to (25), substituting the correct value of $\frac{u_1}{\sqrt{g\,H}}$, and in these equations a term $F_3\,\frac{u_2^2}{2\,g}$ should be added to the left hand to represent the additional loss of head at inflow and in the wheel.

14. *Design of Mixed-flow Fans.*—In the previous designs we have supposed that inflow takes place from both sides, and when a fan is forcing air into closed passages there is no reason why this should not be the case; but when the air is being drawn from passages it is more convenient to erect the fan with inflow from one side, when the area of the eye must be made twice as great, and consequently the radius 1·414 times. The ratio of the external to the internal radius cannot be rigidly fixed. Under these circumstances

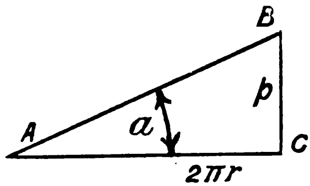


FIG. 10.

a fan of the mixed-flow type is very suitable, and we shall now deal with this class. The Rateau fan is a very good illustration of this type of fan. Tho vanes seize upon the air immediately it enters the eye of the fan, although there is a mouthpiece before the eye whose object is to make the velocity of inflow as uniform as possible over the whole area. The angle θ must therefore vary with the radius, because

$$\cot\theta = \frac{c_2}{u_2} = \frac{R_2\,\omega}{u_2}$$

where ω = angular velocity of wheel in radians $= \frac{2\,\pi}{60} \times$ revs. per minute.

But R_2 varies from zero at the centre to r_2 at the outer radius of the eye. Now, if a spiral of pitch p, fig. 10, and radius $2\pi r$, be traced round a cylinder and the cylinder developed, one turn of the spiral will give us the line A B, whose inclination a to a line perpendicular to the axis gives us

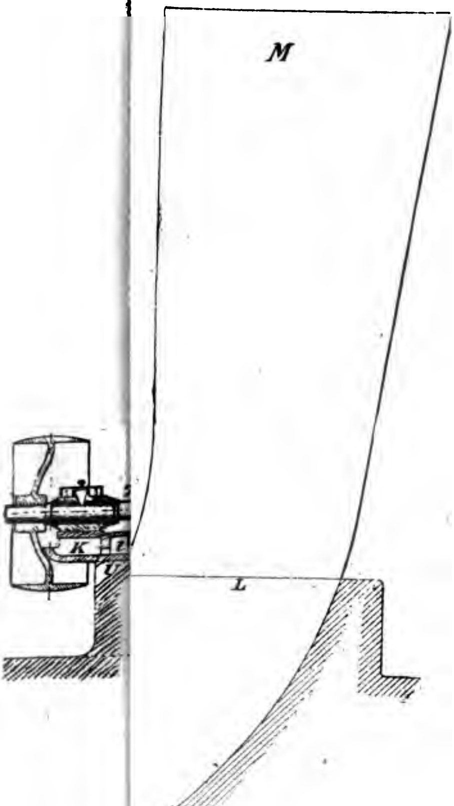
$$\cot a = \frac{2 \pi r}{p}.$$

Now, if a number of these spirals are traced with different values of r, it is clear that the cotangent of the angle increases as r, and the surface generated by joining all these spirals by lines perpendicular to the axis is called a helical or screw surface. It may also be generated by a straight line which passes through and is perpendicular to the axis, and which rises a distance p with uniform velocity while it makes a complete turn with uniform velocity. A straight line is usually employed to form a true helix, but a surface traced by a curve, whose plane contains the axis, and whose outer end is on a cylinder having this axis, this end tracing a spiral on the cylinder, will also trace a surface, any cylindrical section of which will give a spiral in which

$$\cot a = \frac{2 \pi r}{p},$$

and the end of each vane of a mixed-flow fan must be a surface of this description. It should then guide the air round, so that it may flow in planes perpendicular to the axis of the fan.

As the Rateau fan is the best illustration of a mixed-flow fan with which we are acquainted, we shall now describe it and give its proportions (figs. 11, 12). The fan centre consists of a cast-iron wheel A, upon which the vanes are fixed. A is formed by the revolution of the arc of a circle about the axis. At the eye it is conical, and at the periphery it is normal to the axis. In small fans the vanes are placed in the mould when A is cast, and in larger sizes they are held to it by angle irons. The number of vanes is 20 to 24 for small fans, and 24 to 30 for large. The edge $c\,d$ of the vanes is as close to the casing as possible, and $c\,d$ is calcu-



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lated so that the relative velocity of the air between the vanes either remains constant or increases slightly from inflow to outflow. The fan centre is fixed to the end of the shaft, and the air is guided at inflow by the conical end *a f a*, and also by the conical or bell-mouthed inlet Q Q, and experiment shows that at inflow the velocity is nearly uniform over the whole section. The casing contains diffuser and volute, which ends in a conical chimney. The diffuser increases slightly in width from inflow to outflow, and is spiral in form instead of circular, its height increasing in proportion to the angle measured from the radius through C to the bottom of the chimney. This necessitates a volute, whose cross section S increases according to the law :—

$$S = \left(x \frac{\theta_1}{360} + y \frac{\theta_1^2}{360^2}\right) \frac{Q}{V}$$

where v is the velocity at the commencement of its spiral, and θ_1 is the angle in degrees measured from the radius through C. The kinetic energy of the air is thus reduced, and its pressure increased.

It is claimed by the inventor that the above arrangement gives better results than a circular diffuser, and a volute whose section increases in proportion to θ_1. These fans receive the air on one side as a rule. This makes the installation easier, and avoids sharp corners before the eye of the pump. However, the wheel is not then completely balanced, and means have to be taken, such as a collar bearing, to prevent end motion. This lateral pressure is, however, only very small, because care is taken by means of the casing to isolate the atmospheric pressure. In consequence, the re-entry of air at space, which exists between the periphery of the fan and the internal circumference of the diffuser, is prevented, and the pressure behind the fan is lowered much below that of the atmosphere, so that the wheel is only very slightly pressed towards the suction side, and this is sometimes completely balanced by the centrifugal force of the air on the curved cast-iron surface of the fan.

The vanes are of wrought iron, and are stamped in a mould. They have the following advantages: They are very rigid in consequence of their curvature in every direction, and the trajectory of every particle of fluid has an almost constant curvature, which is of great importance in lessening eddies.

Fig. 13 is the geometrical construction for the vanes: K O is the axis of the fan shaft, and circles passing through S, P, Q, R have radii equal to the external radius of the fan, while P R is the breadth of the vane at the tip. Take a point B on the circle S, and through it draw arcs

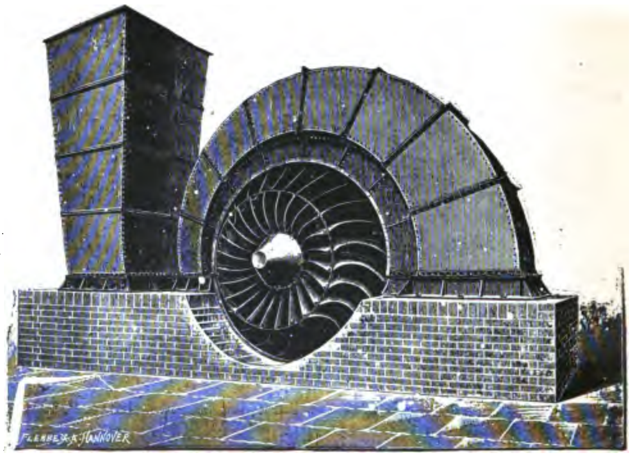


FIG. 12A.

B e, B C; the angle made by the arc B e with the tangent at B is 45 deg. for a fan of type A (see Table I.), and generally is the angle made by the vane at the external radius r_1 with the tangent. The rotation in the figure is supposed to take place from B to the right, so that if vanes curved backwards are used the centre of B e would lie on the opposite side. The circular arc B C makes an angle C B B with the line B B parallel to the axis, such that, if u_2 is the intended velocity of the air at normal speed, and c_1 is the peripheral speed at the extreme radius, then tan C B B $= c_1 / u_2$, and this will ensure that at any point on the arc B e the air will meet the edge of the vane when the fan is running at the orifice for which it is designed. In tracing out the surface of the vane, the arc B e moves with B on the arc BC, and e on the cylindrical surface, with $e x$ as radius and $x x$ as axis; the whole of this surface is, however, not required for the vane. Imagine $a b c d$ to sweep round the axis $k x$ and to cut out a portion; this forms the vane, $a d$ being the part near the

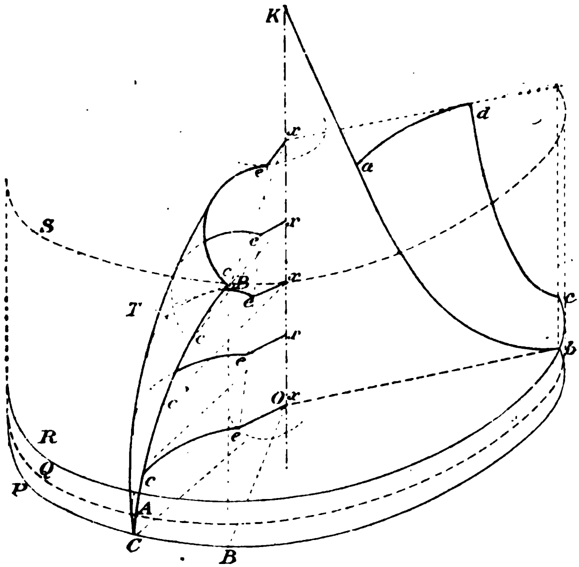


FIG. 13.

eye and $b c$ that near the periphery, while K a forms the cone at the eye. This construction may therefore be used for any type of mixed-flow vane, whatever the angles θ and ϕ have to be. Opposite are given, in Table I., the proportions of these fans, and in fig. 12A is shown a perspective view of a Rateau fan, with part of the casing removed to show the vanes.

To design a fan of this type for a water gauge of 8 in. and

TABLE I.—Proportions of Rateau Fans.

(The radius of the fan is denoted by r_1.)

		Types. A For large volumes and small pressures (see figs. 5, 5a.)	B For moderate volumes and pressures.	C For small volumes and high pressures.
Suction Orifice—				
External radius	r	$\cdot 6\, r_1$	$\cdot 5\, r_1$	$\cdot 4\, r_1$
Radius of cone	ρ	$\cdot 25\, r_1$	$\cdot 25\, r_1$	$\cdot 25\, r_1$
Area of suction orifice	O	r_1^2	$\cdot 6\, r_1^2$	$\cdot 35\, r_1^2$
Vanes—				
Initial inclination at the circumference of the suction orifice with the axis......	β	45°	45°	45°
Final inclination at the circumference of the fan with the rods..................	α	45°	60°	60°
Depth of Vane—				
At inflow	a	$\cdot 40\, r_1$	$\cdot 28\, r_1$	$\cdot 19\, r_1$
At outflow	e	$\cdot 16\, r_1$	$\cdot 08\, r_1$	$\cdot 054\, r_1$
Diffuser—				
Width	e^1	$\cdot 175\, r_1$	$\cdot 09\, r_1$	$\cdot 05\, r_1$
Depth of Spiral—				
Initial	h	$\cdot 10\, r_1$	$\cdot 10\, r_1$	$\cdot 10\, r_1$
Final	H	$\cdot 50\, r_1$	$\cdot 50\, r_1$	$\cdot 50\, r_1$
Inclination of one face to another	γ	4° + 3°	4° + 3°	3° + 2°
Approximate length of the spiral*	λ	$9\cdot 2\, r_1$	$9\cdot 2\, r_1$	$9\cdot 2\, r_1$
Volute—				
Formula giving the sections as functions of the arc of the spiral x †	s	$\cdot 5\, x\,(1 + .8\, x)\, O$	$.45\, x\,(1 + x)\, O$	$40\, x\,(1 + 1\cdot 2\, x)\, O$
Chimney—				
Height ‡	L	5 to 7 r_1	5 to 7 r_1	5 to 7 r_1
Angle of chimney	θ	7°	6°	5°

* The end of the spiral is at a distance r_1 from the point which is on a radius passing through the origin of the spiral.

† x is the ratio of the length of the arc of the spiral from the origin to the point at which the section is to be calculated to the total length of the spiral λ.

x may be calculated as a function of the angle ω between two radii passing through the origin of the spiral, and the point of the spiral where the section is to be calculated by the following formula: $x = \frac{\omega}{480}\left(1 + \frac{\omega}{1980}\right)$, the angle ω being expressed in degrees.

‡ Calculated from the end of the spiral.

180,000 cubic ft. per mm., we shall assume that the velocities of flow u_1, u_2 are $\cdot 5\ \sqrt{g\,\mathrm{H}}$, and since

$$\mathrm{H} = 8 \times \frac{10000}{144}$$

$$\therefore\ u_1 = u_2 = \cdot 5\ \sqrt{32 \times 8} \times \frac{100}{12} = 66{\cdot}6 \text{ ft. per sec.},$$

so that the area O of the suction orifice (Table I.)

$$\mathrm{O} = r_1^2 = \frac{\mathrm{Q}}{u_2} = \frac{3000}{66{\cdot}6} = 45 \text{ sq. ft.},$$

$r_1 = 6{\cdot}71$ ft., $r_2 = \cdot 6\ r_1 = 4{\cdot}02$ ft., and the radius of cone $\rho = 1{\cdot}675$, so that the external diameter of the fan is 13·42 ft. The vanes are curved forwards, so that $\phi = 135$ deg., and 12 vanes are used, so that we may assume the angle of flow practically coincides with the angle of vane.

What is the best number of vanes for a given fan, is a question that we are unable to answer, as no experiments have been published upon this subject, and designers differ widely on the point. Up to a certain point, the greater the number of vanes the higher the water gauge, but a point must be reached at which additional surface friction not only reduces the efficiency but reduces the gauge.

It will be noticed that the width of the vane at outflow, which we denote by b_1, but which in Table I. is given as e, is $\cdot 16\ r_1$, so that the cylindrical surface through which the air leaves the fan at radius r_1 is $2\ \pi \times \cdot 16\ r_1^2$, which is very little greater than r_1^2. The final depth of the spiral H is 3·355 ft., so that the radius at that point is 10·065 ft., and as the tip-speed of the wheel, from (38),

$$c_1 = \cdot 87\ \sqrt{g\,\mathrm{H}}, \qquad \mathrm{N} = 165,$$

$$\therefore\ w_1 = 1{\cdot}37\ \sqrt{g\,\mathrm{H}} = 182{\cdot}5,$$

$$w_2 = w_1 \frac{r_1}{r_2} = \frac{182{\cdot}5}{1{\cdot}5} = 122,$$

and, according to our theory, this is the velocity in the volute if the chimney has a sufficient section to reduce the velocity to a low value. Hence, following our theory, the section of the volute at discharge

$$S = \frac{3000}{122} = 24{\cdot}6 \text{ sq. ft.},$$

while, according to Table I.,

$$s = {\cdot}5\, x\,(1 + {\cdot}8\, x)\, r_1^2,$$

and $x = 1$ when $s = S = {\cdot}5 \times 1{\cdot}8 \times 45 = 40{\cdot}5$ sq. ft. According to our theory the section of the chimney would be

$$S_1 = \frac{Q}{\frac{1}{8}\sqrt{g\,H}} = \frac{3000 \times 12}{200} = 180 \text{ sq. ft.},$$

and, assuming the section of volute and chimney square, the chimney would require to be long enough to alter the side of the square from 4·95 ft. to 13·4 ft., which, with a total taper of 7 deg., would require a chimney whose height was

$$L = \frac{13{\cdot}4 - 4{\cdot}95}{\tan 7^\circ} = \frac{8{\cdot}45}{{\cdot}1228} = 69 \text{ ft., nearly}$$
$$= 10{\cdot}3\, r_1$$

instead of 5 to 7 times r_1 (Table I.). In this case the brake horse power

$$\text{B.H.P.} = \frac{62{\cdot}3 \times 8 \times 180000}{12 \times 33000 \times {\cdot}84} = 270;$$

$$\text{I.H.P.} = \frac{270}{{\cdot}85} = 318.$$

If, however, we assume a diffuser in which $r_3 = 1{\cdot}5\, r_1$, and that $v_4 = \frac{1}{2}\, w_3$, the chimney will be greatly reduced in height. The values of r_1, r_2 are the same as before, but, from (38A),

$$c_1 = {\cdot}885\sqrt{g\,H}, \qquad N = 168,$$
$$w_1 = 1{\cdot}385\sqrt{g\,H},$$
$$v_4 = \tfrac{1}{2}\, w_3 = \tfrac{1}{3}\, w_1 = {\cdot}462\sqrt{g\,H};$$

hence the section of the volute at discharge

$$= \frac{3000}{61 \cdot 6} = 48 \cdot 6 \text{ sq. ft.,}$$

which would form a square whose side is 6·97 ft., so that the length of the chimney would require to be

$$L = \frac{13 \cdot 4 - 6 \cdot 97}{\tan 7^\circ} = \frac{6 \cdot 43}{\cdot 1228} = 52 \cdot 4 \text{ ft.}$$

$$= 7 \cdot 8 \, r_1.$$

Here B.H.P. = 278, I.H.P. = 328.

In the first case, the angle θ that the vanes at the outer radius of the eye make with a tangent to the direction of motion—*i.e.*, with a line perpendicular to both radius and axis—is given by

$$\cot \theta = \frac{c_2}{u_2} = \frac{\cdot 6 \, c_1}{\cdot 5 \sqrt{g \mathrm{H}}} = \frac{\cdot 6 \times \cdot 87}{\cdot 5}$$

$$= 1 \cdot 04$$

and in the second,

$$\cot \theta = \frac{\cdot 6 \times \cdot 885}{\cdot 5} = 1 \cdot 06,$$

so that θ is as nearly as possible 45 deg.

14*a*. *On the Theory of the Spiral Diffuser and Volute.*— Let ρ_3 be the radius of the diffuser at an angle θ from its commencement, and let

$$\rho_3 = r_1 + c\,\theta + a$$

where a and c are constants.

Then
$$\frac{w_3}{w_1} = \frac{r_1}{r_1 + c\,\theta + a}$$

The curve of flow of a particle in the diffuser is an equiangular spiral. Let A, fig. 13*a*, be the point where this particle leaves the wheel, and B where it leaves the diffuser and enters the volute, so that A B is an equiangular spiral.

Let C be the wheel centre, and A C B be called α, and let the line C B be at θ from the radius passing through the commencement of the diffuser. Then

$$\log \frac{\rho_2}{r_1} = K\alpha = \log \frac{r_1 + c\theta + a}{r_1}$$

where K is the cotangent of the angle made by the equiangular spiral with the radius at any point, so that

$$K = \frac{u_1}{w_1}$$

$$\therefore \quad \alpha = \frac{1}{K} \log \frac{r_1 + c\theta + a}{r_1} = \frac{w_1}{u_1} \log \frac{r_1 + c\theta + a}{r_1}.$$

Let S be the section of the volute made by a plane passing through the axis and C B. Then the section S has a quantity

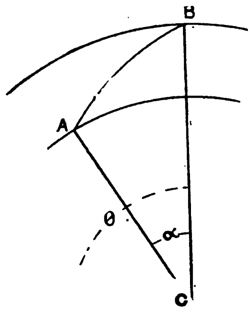


FIG. 13A.

of air passing through it per second, represented by the quantity

$$Q \frac{\theta - \alpha}{2\pi}$$

and we may suppose its velocity to be either w_2 or $\frac{1}{2} w_2$ at this section. Hence

$$S = \frac{Q(c\theta + r_1 + a)}{2\pi r_1 w_1} \left(\theta - \frac{w_1}{u_1} \log \frac{r_1 + c\theta + a}{r_1}\right),$$

or double this quantity.

CHAPTER VI.

15. *On the Variation of Pressure in a Centrifugal Fan.*—To the designer of fans having two inlets this article is of no interest, but it will be found of service to the designer of radial fans or mixed flow fans having one eye. Suppose the fan is acting by suction, and that the velocity of inflow is u_2, then the pressure per square foot measured positive when above the atmosphere, and negative when below, in feet of air is

$$- \mathrm{H} - \frac{u_2^2}{2g}$$

at the inner radius.

At any radius r in the wheel let the absolute velocity be v_5, and let c and w be the velocity of the wheel, and tangential velocity of the air at that radius; then the work done by the wheel from radius r_2 to r is

$$\frac{c\,w}{g} \text{ per lb.,}$$

and this produces an increase of pressure head

$$\frac{c\,w}{g} - \frac{v_5^2}{2\,g} + \frac{u_2^2}{2\,g},$$

so that the pressure head becomes

$$\left(\frac{c\,w}{g} - \frac{v_5^2}{2\,g} + \frac{u_2^2}{2\,g}\right) - \mathrm{H} - \frac{u_2^2}{2\,g}$$

$$= \frac{c\,w}{g} - \frac{v_5^2}{2\,g} - \mathrm{H},$$

so that if ψ is the inclination of the relative direction of flow in the wheel with the direction of motion of the wheel, then the above head

$$= \frac{c^2 - c\,u \cot \psi}{g} - \frac{u^2 + w^2}{2\,g} - \mathrm{H}$$

$$= \frac{1}{2\,g}\left(c^2 - u^2 \operatorname{cosec}^2 \psi - 2\,g\,\mathrm{H}\right)$$

where u and v_s are the radial component and absolute velocity of flow at radius r and correspond to u_1 and v (fig. 5) at radius r_1.

The above neglects friction, and supposes that inflow takes place without shock. At the outer radius r_1 the pressure head becomes

$$\frac{1}{2\,g}\,(c_1^2 - u_1^2 \operatorname{cosec}^2 \phi - 2\,g\,\mathrm{H}).$$

The actual pressure in pounds per square foot may be obtained from the head by multiplying it by ·075. The total pressure on the wheel in feet of air is therefore

$$\int_{r_2}^{r_1} \frac{\pi\,.\,r\,.}{g}\,(c^2 - u^2 \operatorname{cosec}^2 \psi - 2\,g\,\mathrm{H})\,d\,r$$

in radial flow fans, and to find this the wheel may be divided into a number of small rings so that $\pi\,r\,d\,r$ can be obtained for each, and

$$\frac{1}{g}\,(c^2 - u^2 \operatorname{cosec}^2 \psi - 2\,g\,\mathrm{H})$$

can be calculated at the middle of the ring whose mean radius is r and thickness $d\,r$. This will give a result which will be practically correct.

The above assumes that the wheel, although there is only one eye through which air enters, has an opening of equal diameter at the other side. It also assumes that the disc of the wheel is on one side so that there is axial thrust, so that by making the wheel with a disc on each side, end thrust can be entirely avoided, and the end thrust due to suction at the eye comes on the fan casing and not on the wheel. If, however, we are dealing with a mixed-flow fan we must proceed somewhat differently. Our explanation will be clearly understood by reference to fig. 11. We want to find the axial component of the pressures on the cone $a\,f\,a$ and the curved surfaces $a\,b$, $a\,b$ (neglecting vane thicknesses).

Let r_2 be the radius of the eye, then at the point f the head is

$$-\,\mathrm{H} - \frac{1}{2\,g}\left(\frac{\mathrm{Q}}{\pi\,r_2^2}\right)^2,$$

but as we get further to the left on the surface of the cone the velocity of the air increases, and the section of passage should be calculated perpendicular to the mean direction of the flow shown by the arrows.

Let A be this section, then the head is

$$- H - \frac{1}{2g}\left(\frac{Q}{A}\right)^2,$$

and we may suppose the cone divided up into a number of rings, and a central circle and areas projected on to a plane perpendicular to the axis, multiplied by the corresponding head. From the point a to b we have merely to obtain the integral

$$\int_{R_2}^{r_1} \frac{\pi r}{g} (c^2 - u^2 \operatorname{cosec}^2 \psi - 2 g H) \, dr$$

where R_2 is the radius of a, and it must be recollected that $u \operatorname{cosec} \psi$ is the velocity of flow of the air relative to the wheel. It can be more easily estimated by drawing lines perpendicular to the arrow, representing the mean direction of flow, and calculating the areas of the frustra of cones swept out by these. Then, if A is the area swept out by one of these lines,

$$u = \frac{Q}{A_1},$$

and ψ must be measured from a drawing of the vanes if it cannot be calculated.

This pressure can be balanced by putting straight radial vanes at the back of the wheel, so as to set the air in motion at the back, because this reduces the pressure at the back considerably below that which would be produced either by the atmosphere or by a pressure over its whole area equal to that at the circumference. If there are no vanes at the back, and if the air is not put in rotation at the back, this last pressure will be exerted over the whole of the back of the fan. We know of no case in which fans have been thus balanced, but show in fig. 14 a sectional elevation of a centrifugal pump balanced in this manner. This drawing is taken from a paper on "The Balancing of Centrifugal Pumps," by

Mr. J. Richards, read before the Technical Society of the Pacific Coast. In my opinion the holes shown at 0 are a mistake.

Imagine a space, such as that at the back of the wheel, in which a number of radial vanes cause every particle of air to

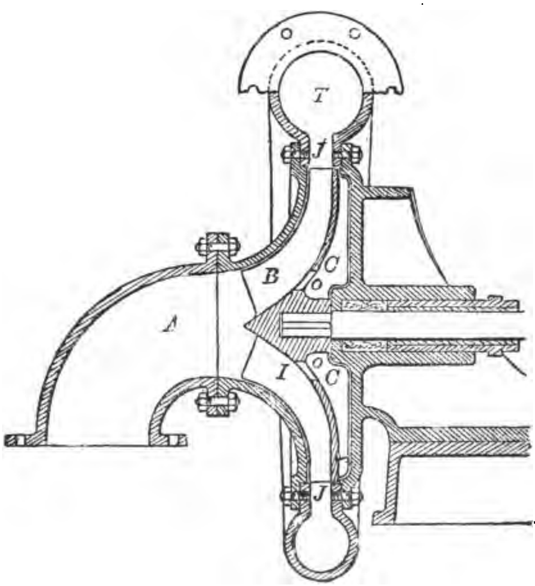


FIG. 14.

rotate at the same angular velocity. Consider a ring of air of mean radius r, and radial thickness $d\,r$, and let the pressure outside be

$$p + \frac{d\,p}{2},$$

and that inside the ring

$$p - \frac{d\,p}{2},$$

and take the axial width as unity. The weight of half the ring is, if D is the weight of 1 cubic foot of air,

$$\pi D \left(r + \frac{dr}{2}\right) dr,$$

and the resultant of the centrifugal force is perpendicular to its diameter, and is

$$F = 2 D \frac{r^2 \omega^2}{g} dr.$$

The resultant air pressure inwards is

$$P = 2\left\{\left(p + \frac{dp}{2}\right)\left(r + \frac{dr}{2}\right) - \left(p - \frac{dp}{2}\right)\left(r - \frac{dr}{2}\right)\right\} - 2 p dr$$
$$= 2 r dp.$$

But P = F for equilibrium, so that

$$2 D \frac{r^2 \omega^2}{g} dr = 2 r dp$$

$$\frac{r \omega^2}{g} = \frac{1}{D} \frac{dp}{dr}.$$

Integrating

$$\frac{r^2 \omega^2}{2 g} = \frac{p}{D} + C$$

where C is the constant of integration, so that, if R_3 be the internal radius at which rotation ceases,

$$\frac{(r_1^2 - R_3^2) \omega^2}{2 g} = \frac{p_1 - P_3}{D} = h_1 - h_3$$

where P_3, h_3 are the pressure per square foot, and head at radius R_3; and at any radius r if P_4, h_4 are pressure and head,

$$\frac{P_4}{D} = h_4 = h_1 - \frac{(r_1^2 - r^2) \omega^2}{2 g}$$

where h_1, in the case of the fan, is

$$h_1 = c_1^2 - u_1^2 \operatorname{cosec}^2 \phi - 2 g H.$$

To calculate the whole pressure on the back of the wheel we must proceed as follows: The pressure head on a ring of mean radius r, and radial thickness $d\,r$, is

$$2\,\pi\,r\,d\,r\left(h_1 - \frac{(r_1^2 - r^2)\,\omega^2}{2\,g}\right),$$

so that the total pressure head between the radii R_3 and r_1 is

$$\int_{R_3}^{r_1} 2\,\pi\,r\,d\,r\left(h_1 - \frac{(r_1^2 - r^2)\,\omega^2}{2\,g}\right)$$

$$= \left(r_1^2 - R_3^2\right)\left(\pi\,h_1 - \frac{\pi\,\omega^2}{4\,g}\left\{r_1^2 - R_3^2\right\}\right).$$

To this we should have to add $h_3\,\pi\,(R_3^2 - R_5^2)$ the pressure on the flat annular end of the boss, R_5 being the radius of the shaft. But if the vanes at the back do not extend to a radius r_1, but to a smaller radius R_1, then the total pressure head on the back of the disc is

$$\pi\,(r_1^2 - R_1^2)\,h_1 + \pi\,(R_1^2 - R_3^2)\left\{h_1 - \frac{\omega^2}{4\,g}(R_1^2 - R_3^2)\right\}$$
$$+ h_3\,\pi\,(R_3^2 - R_5^2);$$

we may note that

$$h_3 = h_1 - \frac{\omega^2}{2\,g}(R_1^2 - R_3^2)$$

in the latter case, and in the former put r_1 for R_1.

15*a*. *On Similar Fans.*—Suppose two fans made from the same drawing but to a different scale so that the dimensions of the one are n times that of the other, and suppose them to be driven so that the velocity of flow through corresponding parts of them is the same. Then the quantity delivered by the one will obviously be n^2 times that by the other, and if the water gauge produced by each is the same, then the orifices will be as n^2 is to 1, and the losses of head due to surface friction are the same because the areas of surface divided by the sections of the fan passages are the same, the roughness of the surfaces of course being supposed to be the same. If the manometric efficiencies are the same the work per pound done by the wheels and the losses due to shock at inflow to them and outflow from these, are the same and therefore the air efficiencies or

$$\eta = \frac{g\,H}{c\ w_1} = \frac{\frac{c_1\,w_1}{g} - L}{\frac{c_1\,w_1}{g}}$$

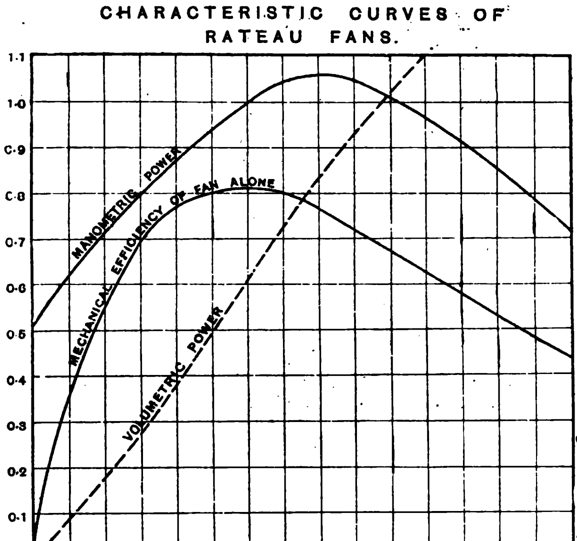


FIG. 14A.

where L is the losses of head, are the same. And we may reasonably suppose the manometric efficiency of the one to equal that of the other because the equation connecting c_1, H and internal velocities depends on the *proportions* of a fan and not on its absolute dimensions, and the proportions are the same in both; hence we have manometric efficiency, and mechanical efficiency are the same for orifices in the ratio of n^2 to 1. Now if we call

$$\frac{Q}{r_1^2 \sqrt{g} H} = O_R$$

the reduced orifice, then a diagram can be drawn with these as abscissæ, and the manometric and mechanical efficiencies as ordinates and we shall get two curves which will apply to fans of a given type but of different sizes. The volumetric efficiences $Q \div c_1 r_1^2$ is obviously the same at equal reduced orifices. In fig. 14*a* are shown characteristic curves of this nature for Rateau fans. We need not enlarge on the assistance such curves should give to the intelligent designer, but he must remember that engine friction must not be included in the mechanical efficiency, and that if there are sharp angles before the eye of the fan which disturb the inflow both mechanical and manometric efficiencies will be much reduced.

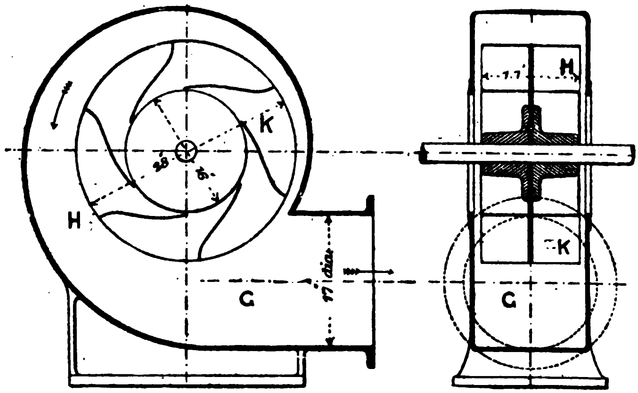
CHAPTER VII.

Experiments on Fans.

16. *Messrs. Heenan and Gilbert's Experiments on Centrifugal Fans.*—The first experiments that we shall describe were made by Mr. Hammersley Heenan, M.I.C.E., and Mr. William Gilbert, Wh.Sch., A.M.I.C.E.* Their object was (1) to determine the best shape of fan blade and fan case in order to secure a minimum expenditure of power when producing any given output of air, *i.e.*, to find the best type of fan; (2) the standard type being selected, to obtain data whereby the proper diameter of the standard fan and its most economical speed could be determined for any required output of air at a given pressure. They came to the conclusion that a fan with a few simple blades gives the best

* Proceedings of Institution of Civil Engineers, vol. cxxiii., Part I.

result, provided the form of blade and dimensions of the casing are designed to suit the kind of work required. Fans of more complex design have, in their opinion, too large an internal resistance to give the highest mechanical efficiency, although they are better for producing high pressures. Careful tests of their measuring instruments were made beforehand. The type of fan tested by them is shown in fig. 15, and consists of a centre or drum H K fitted with about six blades, revolving within a casing. The air enters



Scale, ¼ inch = 1 foot.

28-INCH BLAST FAN.

FIG. 15.

through the centre of the sides of the drum and is discharged at the outlet G; we must here remark, in parenthesis, that the design of both casing and wheel appears to be faulty. The wheel has parallel sides and hence the sudden change of direction at inflow from axial to radial must cause eddies, and some of the air is discharged from the wheel against the upper curved surface of the outlet while the flow from the outlet must be anything but uniform over the whole section as indeed was proved by experiment.

Suppose such a fan to be running at a constant speed, taking air from the atmosphere and discharging through a delivery tube H K, fig. 16, having an outlet at K, the area of which can be varied. Let the fan centre be 28 in. diameter and the delivery tube 17 in. diameter. Let there be two water gauges, A and B, one of the branches of each of which is connected to a pipe soldered into the side of the delivery tube in the gauge A; the other branch is attached

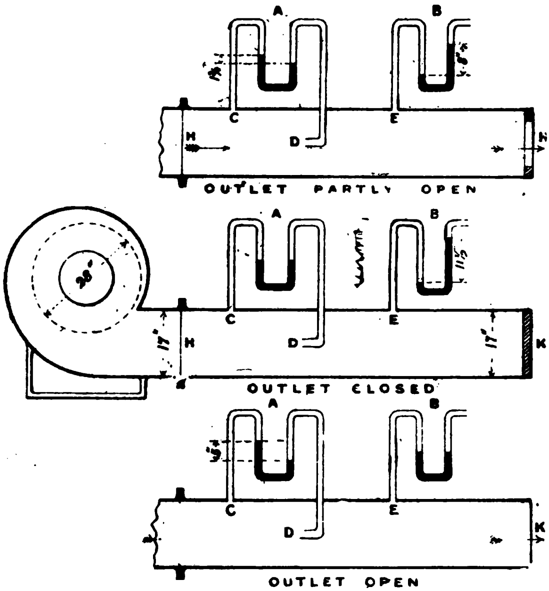
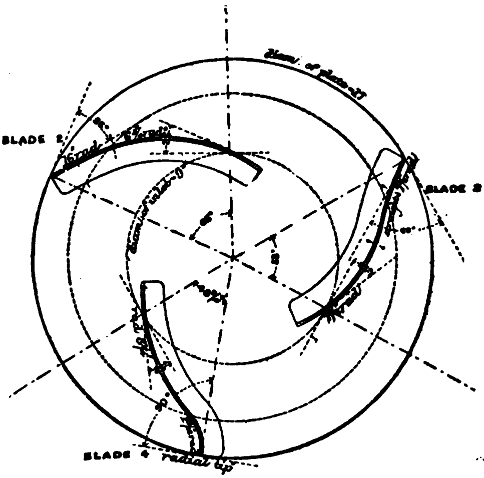


FIG. 16.

to a pipe passing to the centre of the delivery tube, the end being suitably bent to face the stream. The pressure in the pipe C (inductive action being prevented) will be that due to the compression of the air only, but the pressure in D will be that due to the pressure and velocity combined. Hence the gauge A, which indicates the difference between the pressures in the pipes C and D, will record the pressure due to velocity only. The compression of the air can be measured by the second gauge B. First, if the outlet be closed so that no air is delivered, the gauge A will remain at zero, since there is no flow of air through the tube, but the second gauge B will indicate a considerable compression, about $11\frac{1}{2}$ in. of water, if the tip speed of the fan is 12,000 ft. per minute. Next let the end of the delivery tube be opened to give, say, half the area of the outlet K. The fan now



Scale, ¼ full size.

FIG. 17.

passes a considerable quantity of air, about 8,000 ft. per minute, and on account of this flow, the velocity gauge will indicate nearly $1\frac{3}{4}$ in. of water. The compression, as shown by the gauge B, will have fallen to 8 in. During the time the outlet of the delivery tube was closed, with no air being delivered the efficiency was of course zero. But when the fan is passing 8,000 cubic feet of air, under a compression of 8 in. of water, the efficiency reckoned on the compression alone will be about 66 per cent, 15 horse power being required to drive the fan. When the outlet of the delivery tube is fully opened, the fan delivers freely to the atmosphere, and gauge B shows that the air is under no compression whatever, but the amount of air has increased to about 13,700 cubic feet per minute, and the passage of this air through the delivery tube shows a velocity on A of nearly 5 in. of water. Since the air is not compressed but merely expelled at atmospheric pressure the efficiency reckoned on the compression is now zero. There is necessarily a delivery intermediate between no delivery and no compression, at this tip speed of 12,000 ft. per minute, for which the mechanical efficiency, reckoned on the compression produced on the air, is a maximum.

Messrs. Heenan and Gilbert preferred to draw a diagram in which the abscissæ were cubic feet of air per minute for a fixed tip speed and the ordinates efficiencies, and water gauges at that speed; a third curve was drawn showing the brake horse power required by the fan as an ordinate, and a fourth curve dotted shows the total efficiency, the kinetic energy in the air discharged being added to the work done in compressing the air. This is sometimes called dynamic efficiency and is misleading, except in cases in which the fan can be fitted with an expanding chimney, and then the kinetic energy left in the air at its mouth must be deducted. A fan that has a high dynamic efficiency may really be one of very low real efficiency, and if quantity of air is of more consequence than the smallness of brake horse power then the volumetric efficiency should be given, and not an imaginary mechanical efficiency which is not attainable. The horse power of the fan considering compression alone was calculated by the formula,

$$\text{Fan horse power} = \frac{5{\cdot}2\, h\, Q_1}{33000} = \frac{h\, Q_1}{6352},$$

where h = water gauge in inches and Q = cubic feet per minute the pressure of 1 in. of water being equal to 5·2 lb. per square foot. Three forms of blade were tested shown fig. 17, numbered 2, 3, and 4 making angles of 35 deg., 60 deg., and 90 deg., with tangents to the outer circumference, and 25 deg. with tangents to the inner circumference, the fan centre being 17 in. in diameter and 8 in. wide; figs. 18, 19, 20, give curves of efficiency, water gauge and brake horse power for each of these blades for a constant tip speed of 12,000 ft. per minute. Blade No. 4 give the best result, although not much superior to blade No. 3. The arrangement of apparatus for measuring the brake horse power, volume of air, compression and speed are shown in fig. 21. The fan is driven from the counter shafting B B, which derives its motion from a spherical steam engine C. The outlet of the fan is connected by a short circular iron delivery tube with a boiler flue E E, 30 in. in diameter and 18 ft. long. At the centre of the flue a partition F is fitted, to which can be attached a series of plates having circular orifices of various sizes, varying between 4¾ in. and 18 in. diameter. A well-cut outlet of known diameter is placed at the end of the flue G, where the velocity of the air was measured by an anemometer. This opening was made much larger in diameter than the outlet of the fan, in order to avoid injuring the anemometer by a violent current of air. This outlet and the levers whereby the anemometer was moved over all portions of the outlet, the instrument being kept truly perpendicular to the axis of the flue, are shown in the end elevation. The pulley H driving the fan was not keyed to the shaft, but was driven by it through the Emerson power scale, a form of transmission dynamometer in which the moment of the driving effort is balanced, through a system of levers by a pendulum moving over a graduated scale. A speed counter records the revolutions of the shaft. A tachometer K, coupled to the countershafting B B by a spiral spring, enabled the speed of that shafting to be regulated, and the proper tip speed to be approximately maintained by the man in charge of the engine. A small band counter, carried in a sliding frame L, and applied when necessary to the fan spindle, enabled the exact number of revolutions per minute of the fan to be obtained. Measurements of the pressure and velocity of the air-stream were taken at the section M M of the delivery tube. The velocity varied greatly in different positions on the same cross-section of the tube. Readings were taken at several points in a cross-section by means of two gauges, each of which could be traversed over a diameter at right angles to the other. It was afterwards found unnecessary to measure the velocity, since it can be readily calculated when the discharge, as measured by the anemometer, is known; but at the same time the velocity, as measured by the velocity gauge, gives a check on the anemometer readings. If the cross-section of the delivery tube be divided into a number of imaginary areas and the square root of the mean gauge reading for each area be multiplied by that area and by a suitable constant, then these results added together give the total discharge of the fan. The annexed table illustrates the variation of velocity referred to; it gives the gauge-reading due to the velocity in four positions along a diameter of the cross-section of the delivery tube, where the gauge was fitted, as taken simultaneously.

Position.	Gauge reading due to velocity.	Air velocity, feet per second.	Distance from centre of tube.	Remarks.
			Inches.	
1	3·20	118	+ 3·1	Diameter of delivery tube, 8·9 in.
2	2·88	112	+ 1·1	
3	1·85	90	- 1·1	
4	1·70	86	- 3·1	

This, we think, is a proof of imperfect design of the fan, because if well designed there is no reason why the velocity should not be almost uniform over the whole section of the tube; if anything, it should be greater at the centre than the sides. It will be noticed in the drawing of a similar type of fan, fig. 15, that some of the discharge from the fan is thrown against the upper surface of the discharge pipe, instead of the whole stream flowing out parallel to the axis of the tube, as it should do. The measurement of the compression presented some difficulty, owing to the fact that the air flowing across the end of the side gauge caused a large amount of induction; a vacuum being often recorded where a pressure was known to exist. The authors were indebted to Professor W. C. Unwin for the information that a plate placed across the end of the tube would prevent this inductive action. The form of side gauge used for measuring

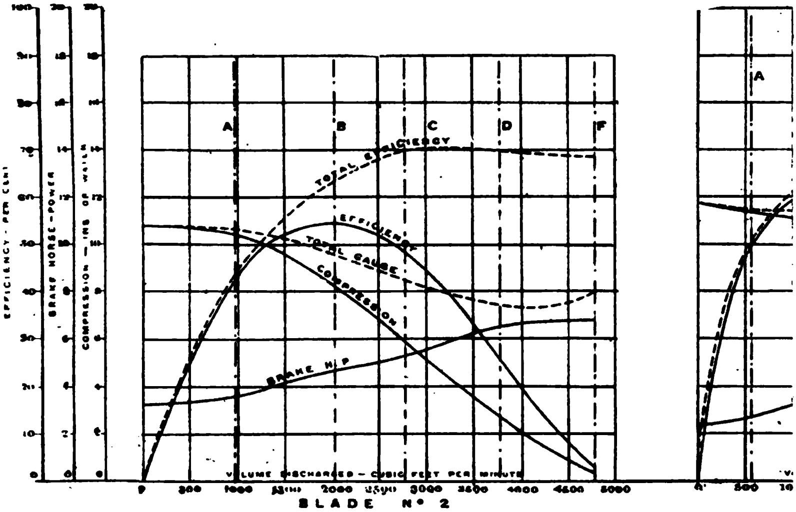
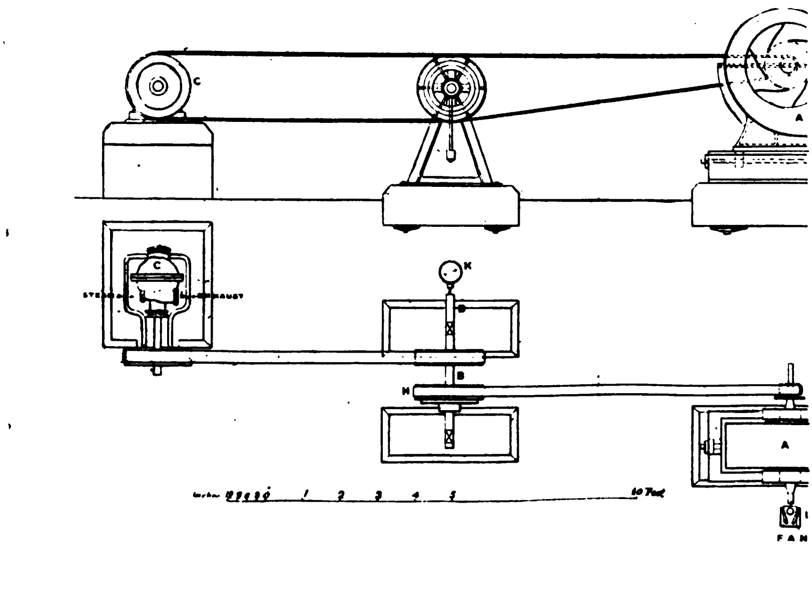

EFFICIENCY - PER CENT
BRAKE HORSE-POWER
COMPRESSION - INS OF WATER
A
B
C
D
F
TOTAL EFFICIENCY
EFFICIENCY
TOTAL GAUGE
COMPRESSION
BRAKE H.P.
VOLUME DISCHARGED - CUBIC FEET PER MINUTE
BLADE Nº 2

FIG. 18.


C
A
K
B
H
STEAM
EXHAUST
10 Feet
FAN

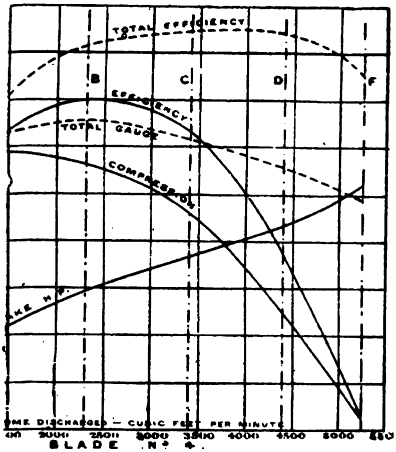
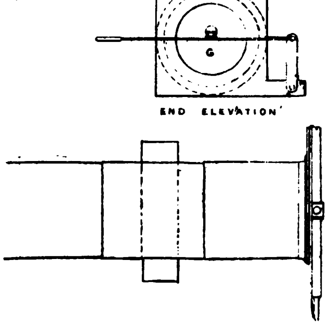

TOTAL EFFICIENCY
B
C
D
F
EFFICIENCY
TOTAL GAUGE
COMPRESSION
VOLUME DISCHARGED — CUBIC FEET PER MINUTE
BLADE No 4.

FIG. 20.


E
G
G
END ELEVATION

CF

he static pressure is shown in fig. 38. It was tested by Messrs. Heenan and Gilbert, and found to give very good results. To draw the characteristic curves from the experimental results, seven resistance plates, A, B. C, D, E, F, with graded circular orifices, were arranged to fasten on to the centre F of the boiler flue. Two observations were taken with each plate of the discharge, the compression and horse power supplied to the fan at or near tip speeds of 5,000 ft., 6,000 ft., and 12,000 ft. per minute. The authors verified the following laws :—

1. The air discharge varies as the speed.
2. The gauge reading varies as $(\text{speed})^2$.
3. The B.H.P. varies as the $(\text{speed})^3$.

} For a constant resistance.

We have already given these laws in preceding articles. Curves were then drawn with cubic feet per minute at a given tip speed as abscissæ, and with ordinates, water gauge,

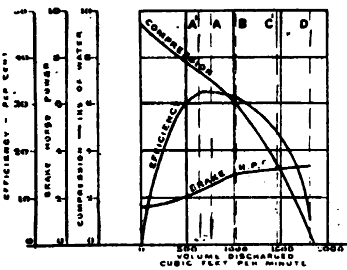


FIG. 22.

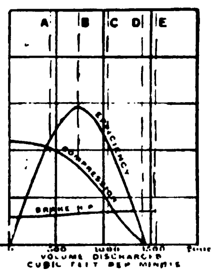


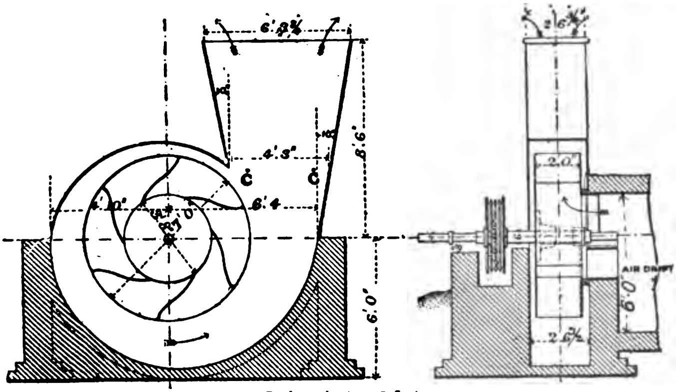
FIG. 23.

brake horse power, mechanical efficiency, static and dynamic, and dynamic water gauge, at the same tip speed. The dynamic water gauge is the sum of the static gauge, and the gauge due to velocity. These curves are shown in figs. 18, 19, 20, for tip speeds of 12,000 ft. per minute. All the recorded tests of Heenan fans were made with centres having parallel sides, and with the proper clearance in the case to allow the air to flow uniformly from the fan centre at all portions of its circumference. With the same apparatus tests were made on several fans of various types.

Fig. 22 shows the characteristics obtained at a tip speed of 12,000 ft. per minute from a fan 16 in. diameter, having tapering side plates. The diameter of the inlet was $5\frac{7}{8}$ in., the widths of the fan centre at inlet and outlet being $7\frac{1}{4}$ in. and $1\frac{1}{8}$ in. respectively. This fan ran in a concentric case, the clearance being $1\frac{1}{8}$ in. (which in our opinion made the comparison of tapering *versus* parallel sides worthless, as the preceding theory very clearly shows that a well-designed volute is necessary to obtain high efficiency and a suitable water gauge). The compression obtained with closed outlet was 9·4 in., against 11·2 in. with blade No. 4, fig. 17. Further, the compression fell off very rapidly with increase of output, so that the working compression would not be more than 8 in. The efficiency was also low (as might reasonably be expected). Fig. 23 shows the characteristic obtained at 12,000 ft. per minute from a fan centre 12 in. diameter, the maximum width being $2\frac{3}{4}$ in. The centre rotated in a whirlpool chamber of 23 in. diameter. The centre was of cast iron, and the tip angle of the blades was 30 deg. Of course, the efficiency measured, which was less than 30 per cent, would not be representative for so small a fan, but the maximum compression did not exceed 4·4 in. The effect of a whirlpool chamber (usually called a diffuser) is to produce a very quiet running fan.

17. *Test of a Mine Ventilating Fan.*—This fan was supplied to the Parkend Colliery Company, South Wales, by Messrs. Heenan and Gilbert. The fan, in connection with the approach tunnel and ventilating shaft of the mine, is shown in fig. 24. The wheel is 7 ft. diameter and 2 ft. wide. The upper portion of the case and evasé chimney is built up of wrought-iron plates, the lower portion being formed in brickwork $4\frac{1}{2}$ in. thick. The fan was driven by a horizontal non-condensing engine, the cylinder being $12\frac{1}{8}$ in. diameter and $17\frac{3}{4}$ in. stroke. Cotton ropes were used for driving, two only being in operation at the time of the experiment. The engine was an old one, and was not supplied with the fan. The boiler pressure averaged 40 lb. per inch. To provide a variable resistance for the fan, three 9 in. by 3 in. planks were placed across the mouth of the air drift, and boards nailed to these planks restricted the flow

of air to the fan more or less as required. The folding doors at the top of the ventilating shaft were open during the whole of the test. A tachometer, driven by a belt from the fan shaft, enabled the approximate speed of the fan to be judged and regulated by the man in charge of the engine; the number of revolutions in a two-minute reading being obtained by a hand-counter held to the fan shaft. The engine speed was obtained by a small counter applied to the shaft in the same manner. The air discharge was measured



Scale, ⅛ inch = 1 foot.

7-FOOT HEENAN MINE-FAN, PARKEND COLLIERIES.

FIG. 24.

by an anemometer held at the top of the fan chimney, a staging being erected for the purpose. The area of the top of the chimney was divided into eight equal rectangles by means of wires tightly stretched across, and the anemometer, attached to a small iron tube, was held for a quarter of a minute in each division. In this fan the flow of air was fairly uniform over the whole of the outlet area of the chimney, but in some cases, where the fan was run slowly for experimental purposes, guide vanes had to be fitted in the base of the chimney to secure the result mentioned. Four degrees of opening were arranged at the adjustable orifice, and for each of these readings were taken with tip speeds of 4,000 ft., 5,000 ft., 6,000 ft., 8,000 ft., and 9,000 ft. per minute. The duration of each reading was two minutes, and it was taken twice. The observer with the watch was stationed in the engine-room, and signalled by an electric bell to the observers at the anemometer and fan counter. Indicator diagrams were taken from the ends of the engine cylinder during each reading. The vacuum produced by the fan was measured by a side gauge, placed in the air drift close to the fan inlet, and a pipe led from this tip to a water gauge placed on a table outside. (In our opinion this is one of the reasons for the apparently high efficiency obtained by the fan, which at a tip speed of 9,000 ft. per minute reached 70 per cent.) We have already stated in Section 5 that if the manometer is placed in a strong current of air the water gauge will be increased, and that at inflow to the fan this will not be the correct gauge. When the fan was running at a tip speed of 9,000 ft. per minute the discharge from the fan at the efficiency of 70 per cent was 14 cubic feet per square inch of diametral wheel section per minute, so that the flow into the eye, even assuming it to be uniform and neglecting the obstruction of the bearing, was at a velocity of

$$\frac{14}{60} \times \frac{24 \times 84}{\frac{\pi}{4} \times (3\frac{2}{3})^2} = 44{\cdot}6,$$

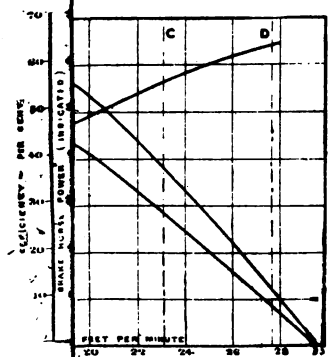
because the eye was $3\frac{2}{3}$ ft. diameter and the centre section 24×84 in. This velocity would increase the water gauge by

$$\frac{(44{\cdot}6)^2}{64} \times \frac{144}{10000} = {\cdot}4475 \text{ in.}$$

It was 5·6 in., and should therefore be reduced to at least 5·16 in., so that the efficiency could not be more than

$$\frac{5{\cdot}16}{5{\cdot}6} \times 70 = 64{\cdot}5 \text{ per cent,}$$

(even assuming that the anemometer did not exaggerate the discharge, which it invariably does.) Three side gauges



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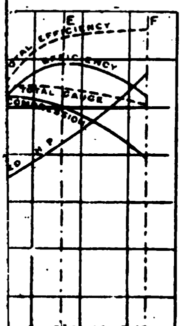


EN AN FAN
00 FT PER MINUTE

C C, fig. 24, were also placed at the root of the chimney, so that the vacuum produced, and consequently the efficiency of the chimney, could be determined. It was found that the vacuum was practically the same at all three, so that only one was read. Figs. 25 to 28 give the characteristic curves obtained from this test. The dotted lines *e e* correspond with the resistance offered when the choking boards were removed and the fan took air from the mine only. The maximum efficiency increases with the speed, and is 67·2 per cent at a tip speed of 8,000 ft. per minute, and 70·3 per cent at

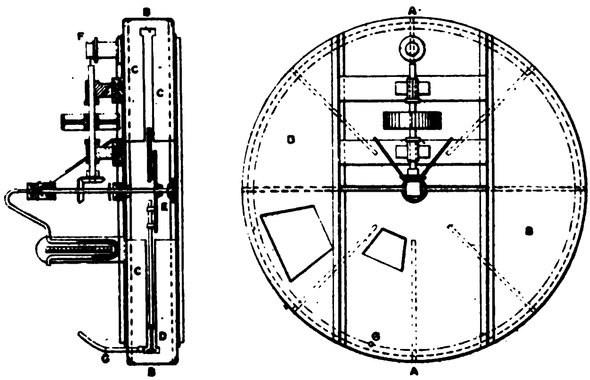


FIG. 32.

9,000 ft. per minute.. The fan was designed to pass 20,000 cubic feet per minute with a water guage of 3½ in., at a speed of 300 revolutions. At 7,000 fr. per minute and 318 revolutions, the water gauge was 3·45 and the discharge 23,150 cubic feet, so that the fan is amply large enough for the work.

18. *Test of Water-gauge Tips.*—The apparatus by means of which the accuracy of the tips used for the measurement of air pressure and velocity was tested is shown in fig. 32. To test the facing gauges, or Pitot tubes, the following method was adopted: The tip was moved at a known velocity through air at rest, and provision was made whereby

a water gauge recorded the pressure thus set up. B B is a wrought-iron circular tank 7 ft. in diameter, and divided into eight compartments by vertical dashboards C. The tip to be tested was screwed into a horizontal pipe D E, attached to and rotated by the hollow vertical spindle of the apparatus. The top bearing of this vertical spindle was formed in a hardwood block, a stuffing box being used in connection with it. From the upper portion of the block a pipe led to the water gauge. Free communication was thus established between the tip under test and the recording water gauge. The vertical spindle was driven from a countershaft by a belt pulley and suitable gearing, the revolutions per minute being recorded by the hand counter F. The tip under test was arranged to describe a circle 20 ft. in circumference. The object of the dashboards being to prevent motion of the air in the tank, the openings in them through which the tip and pipe passed were made as small as possible. A facing gauge G was inserted into the tank, just clear of the path of the revolving tip. No reading was detected on this gauge, so that the velocity of the air in the tank could never have exceeded 10 ft. per second, corresponding with a reading of $\frac{1}{44}$th in. Velocities lower than this could not be read on an ordinary facing gauge. The tip speeds used reached 200 ft. per second.

We think that a few words on the theory of the Pitot tube will not be out of place here. Suppose a fluid in motion impinges upon a plane surface set at right angles to it, and flows away in a direction perpendicular to its original direction; then, since the change of the momentum is equal to the impulse of the external forces (Newton's Second Law of Motion),

$$t \frac{W v}{g} = P t$$

where W is the weight of the fluid impinging on the plate per second, v is its velocity, t the time in seconds, and P the force produced;

$$\therefore \frac{W}{g} v = P;$$

and if A is the section of the jet

$$D \frac{A v^2}{g} = P, \text{ or } \frac{P}{D A} = \frac{v^2}{g} = \frac{p}{D} = H,$$

where p is the pressure per square foot, D is the density, and H is the head equivalent to the pressure p. Hence we might at first imagine that the reading of the Pitot tube would be v^2/g, instead of $v^2/2g$ in feet of air; but the air deviated by the shock upon the end of the Pitot tube does not lose the whole of its momentum parallel to the axis of the tube, so that in reality

$$H = m \frac{v^2}{g}$$

where m is a coefficient smaller than unity, depending probably upon the section of the orifice and of that of the conduit in which it is situated. When the orifice is small compared with the conduit, experiment shows that $m = \frac{1}{2}$ very nearly. If h is the water gauge in inches,

$$h = \frac{12 v^2}{2 g} \frac{D}{\rho} = \frac{12 \times 39{\cdot}8 \text{ B} v^2}{64{\cdot}4 \times 62{\cdot}3 \times 29{\cdot}92 \theta}$$

where ρ = weight of 1 cubic foot of water,

$$= \frac{B v^2}{251 \theta} \text{ for dry air} \quad . \quad . \quad . \quad . \quad . \quad (49)$$

where B is the height of the barometer in inches of mercury, and θ is the absolute temperature in Fahrenheit degrees, or

$$\theta = F + 461;$$

so that if we put B = 29·9 and F = 62 deg.,

$$h = \frac{v^2}{4390} \text{ in. of water.}$$

To test a tip the water gauge connected with the revolving tube is observed, and the reading compared with that calculated from the equation

$$h = \frac{12 v^2 D}{2 g \rho},$$

the correct values of D and ρ being used; a correction must, however, be introduced for the vacuum produced by the centrifugal force of the air in the revolving horizontal tube D E. This vacuum we have already shown in section 15 to be $D v^2/2 g$ at the centre; for, in a mass of air rotating, we proved that

$$\frac{r^2 \omega^2}{2 g} = \frac{p}{D} + C,$$

so that if r_1 be the external radius, and

$$v = r_1 \omega,$$

$$\frac{v^2}{2 g} = \frac{p_1 - p_2}{D}$$

where $p_1 - p_2$ is the difference of pressure between the centre and radius r_1. Hence if the tip measures the pressure due to the air velocity correctly, there should be no reading on the water gauge for any speed, as the vacuum due to the centrifugal force just balances the pressure due to the velocity of the moving tip against the air. As a matter of fact, small readings were observed on the manometer, but these were accounted for by supposing that the air in the tank was drawn round by the rotation of the tip, and that the maximum velocities of the air were—for gauge, fig. 33, 2·7 ft. per second; for gauge, fig. 34, 5·9 ft. per second; and for gauge, fig. 35, 5·4 ft. per second; so that the velocity of the air may be calculated by formula (49) The table giving these experiments will be found in Appendix II. of Messrs. Heenan and Gilbert's paper.

In testing side-gauge tips it is clear that the water-gauge reading will be the vacuum due to the centrifugal force of the air in the rotating tube plus the vacuum produced by the inductive action of the air flowing across the face of the tip. It is the amount of the latter action that is to be determined by experiment. The results of tests on three forms of tips used as side gauges are given in the table below. Experiments Nos. 604 and 605 were made on the side tips, figs. 36 and 37. The vacuum recorded in the revolving tube is given in column 3, and in column 4 is shown the calculated vacuum that would be produced in the revolving tube by the action of centrifugal force. The difference between these readings, as given in column 5, is the vacuum produced on the tube by inductive action at each speed. It follows, therefore, as tabulated in column 6, that if these tips be used to measure the pressure in a stream of air flowing through a pipe, by placing them at right angles to the direction of flow, the pressure recorded

Fig. 33.

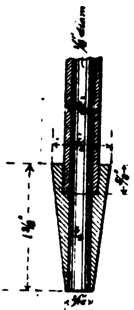


Fig. 34.

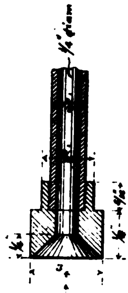
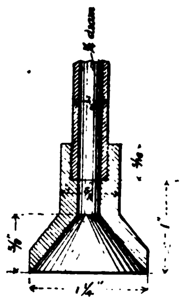


Fig. 35.



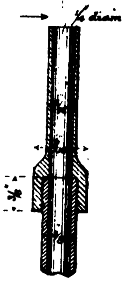


Fig. 36.

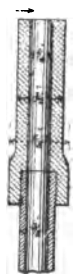


Fig. 37.

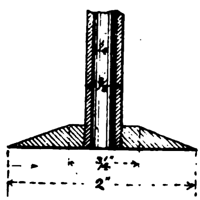


Fig. 38.

will be less than the correct amount by a quantity equal to about 45 per cent of the gauge reading, which would represent the velocity of the air stream in question, column 6. Small differences in the shape of the tips might considerably affect the result. Experiment No. 606 was made on the standard side gauge adopted, fig. 38. The vacuum recorded in column 5 would be accounted for by an air velocity in the tank itself of about the same amount as that produced by the bell-mouthed facing gauges, which caused air velocities in the tank varying between ·66 ft. and

TEST OF SIDE-GAUGE TIPS.

No. of experiment.	Tip speed in feet per second.	Water-gauge reading.	Calculated vacuum due to tip speed	Vacuum due to induction.	Gauge reading due to induction. Per cent.	Remarks.
604	68·8	1·59	1·07	·52	49 0	
	109·5	4·03	2·73	1·30	48·0	Thin side tip, fig. 36. Temperature Fah., 53·2. Barometer, 29·47. $h = \frac{v^2}{4895}$
	137·2	6·18	4·27	1·91	45·0	
	160·0	8·50	5·82	2·68	46·0	
	190·4	11·66	8·24	3·42	42·0	
605	81·8	2·24	1·51	·73	48·0	
	99·8	3·20	2·26	·94	42·0	Medium side tip, fig. 37. Temperature Fah. = 57. Barometer = 29·63. $h = \frac{v^2}{4400}$
	113·2	4·22	2·90	1·32	46·0	
	136·8	6·40	4·25	2·15	51·0	
	167·8	9·65	6·38	3·27	51·0	
606	69·3	1·13	1·09	·04	3·7	
	81·0	1·58	1·49	·09	6·0	
	104·0	2·58	2 46	·12	4·9	Standard side gauge, fig. 38. Temperature Fah. 44 deg. Barometer = 28·89. $h = \frac{v^2}{4400}$
	127·3	3·90	3·68	·22	6·0	
	151·0	5·57	5·19	·38	7·3	
	187·0	8·48	7·95	·53	6·8	

5·9 ft. per second. It appears reasonable to assign the small reading obtained to that cause, and to assume that the inductive action is completely neutralised by this gauge. For low velocities, such as are found in mine drifts, the form of gauge is unimportant, since there is practically no indication on the gauge due to velocity, or any inductive action to be guarded against; but in the case of blast or cupola fans the conditions are very different, and a correct side gauge is essential. (In our opinion the above is the most valuable part of Messrs. Heenan and Gilbert's paper.)

CHAPTER VIII.

19. *Experiments on Centrifugal Fans by Bryan Donkin, M.I.C.E.**—These experiments were made on small fans, but are as valuable as, if not more so than, any other experiments made on fans, as it is extremely probable that the quantities of air recorded are correct. These trials were made in '93—'94. Mr. Donkin found that when the passages remained the same the discharge varied as the speed. In each set of experiments upon a given type of fan the quantity of air passing was varied, and a change in the pressure was produced by throttling the flow in the delivery pipe at some distance from the fan, and allowing the air to pass successively in each experiment through wove wire of 3, 8, 30, and 50 meshes to an inch. The "equivalent orifice" was also varied by the insertion of one to four pieces of perforated zinc superposed. The wove wire of 3 and 8 meshes to an inch gave respectively by calculation an effective area of 80 per cent, and 56 per cent of the area of the pipe. One piece of perforated zinc gave an area of 40 per cent. Experiments under these several conditions, as well as with no baffle, were made upon each fan, the end of the delivery pipe being open to the atmosphere in all cases. This end was, in a final experiment, completely stopped, and the air, instead of passing through the fan, was churned up inside it. The I.H.P. was taken in each case, as well as the pressure of the air and the speed of the fan. About ten experiments were made on each fan, each occupying about 15 minutes after all conditions had become constant, and as a similar series of experiments were made upon each, with the same pipe and apparatus, the results are comparable.

* Proceedings of the Institution of Civil Engineers, vol. cxii.

Eleven different types of fan were tested, of diameters varying between 16 in. and 25½ in. The number and shape of the vanes differed considerably, figs. 39 and 40. Each fan was driven by a strap from the same steam engine, which was indicated to give the power absorbed. The I.H.P. required to drive the engine at different speeds was accurately known, and was in each case deducted from the total I.H.P. A large quantity of air is required at low pressure in some cases, and in others a small quantity at high pressure. The volume of air passing at the maximum pressure, with a given speed of the fan and equivalent orifice, was

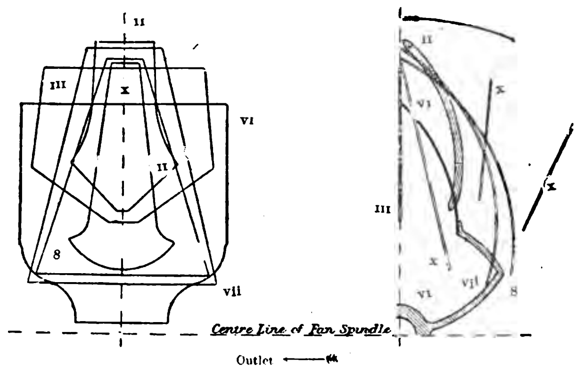


FIG. 39.

determined in the experiments. The terms volumetric, and manometric (or pressure) efficiencies, are explained in Sections 12 and 10, and are—

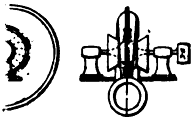
$$\delta = \frac{Q}{c_1 r_1^2} \qquad (46)$$

$$M = \frac{g H}{c_1^2} \qquad (25a)$$

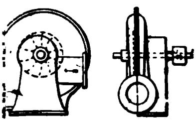
and the mechanical efficiency of the fan itself is

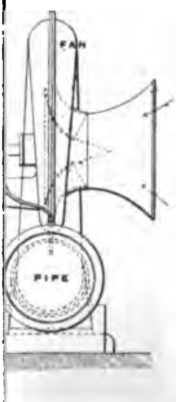
$$\epsilon = \frac{62{\cdot}3\, Q h}{12 \times 550 \times \text{B.H.P.}}$$

N° V.



N° XI.





CF

where Q is the discharge in cubic feet per second and B.H.P. is the horse power supplied to the fan shaft. The number of vanes of each fan, together with their shape and direction of curvature, are given in Table III. The casing in which the vanes revolved differed considerably in shape, but was always of cast iron. The vanes revolved in some cases with the concave, in some with the convex, and in others with the flat side to the outlet. They were set sometimes radially and

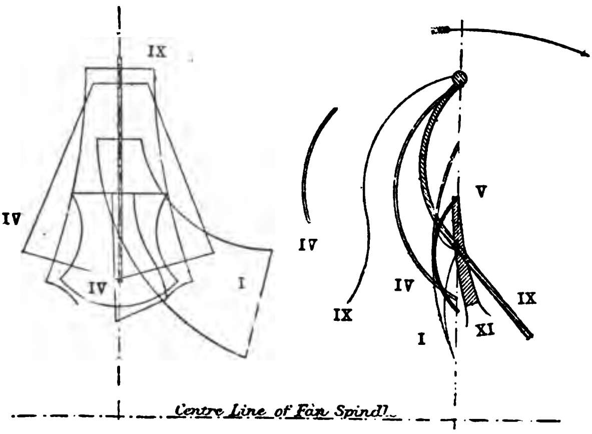
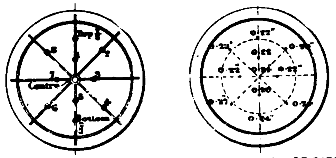


FIG. 40.

sometimes inclined to the centre of the shaft. The fan was in some cases driven with the blades revolving in the opposite direction to that indicated by the makers, and an increase in delivery resulted, although other conditions remained the same. A drawing of each fan is given in fig. 41. The maximum efficiency or the best experiment on each is given in Table III., and the volumetric, manometric (or pressure), and mechanical efficiencies are represented graphically in figs. 43, 44, and 45. As regards possible errors, the speeds, pressures, and quantities of air are probably correct to within 3 and 4 per cent. Table IV. gives the



END-VIEW OF PIPE AT Z SHOWING POINTS AT WHICH PRESSURES WERE TAKEN.

CROSS-SECTION OF PIPE SHOWING PRESSURES AT THE 11 POSITIONS IN INCHES OF WATER

FIG. 42A.

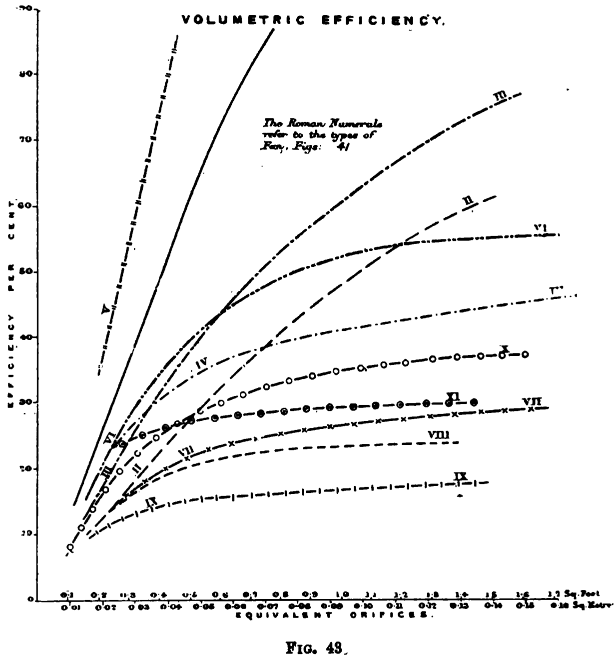


FIG. 43.

TABLE II.—EXPERIMENTS ON FAN NO. 1. TWENTY WROUGHT-IRON VANES RUNNING CONCAVE TO THE OUTLET. ONE OUTLET ONLY.

Revolutions of fan per minute.	Slip of strap per cent.	Static pressure at A in inches of water.	Dynamic pressure at B in inches of water.	Velocity of the air at Z in feet per minute.	Quantity of air delivered in cubic feet per minute.	Temperature of air at the end of the pipe by wet and dry thermometers in deg. Fah.	Barometric pressure in inches of mercury.	Velocity of vane tips in feet per second.	Horse power absorbed by the fan itself.	Theoretical horse power required.	Mechanical efficiency per cent.	Pressure efficiency per cent.	Volumetric efficiency per cent.	Orifice $\frac{Q}{\sqrt{g}\,H}$ in sq. ft.	Equivalent orifice in square feet.	Conditions—Baffles used.
847	4·8	$\frac{3}{4}$ vac.	$\frac{5}{8}$	3341	3560	61 wet, 76 dry	29·8	72·6	2·98	·36	12	26 9	121·3	1·57	1·71	Free discharge.
857	4·6	$\frac{1}{2}$ vac.	$\frac{13}{16}$	3319	3536	61 wet, 75 dry	30·2	73·6	2·67	·46	17·1	33·6	119·0	1·39	1·52	One piece of 3-hole per inch wove wire.
887	3·4	$\frac{1}{8}$ vac.	$1\frac{1}{8}$	3100	3303	62 wet, $76\frac{1}{2}$ dry	30 2	76·1	2·73	·58	21·3	42·8	107·5	1·11	1·20	One piece of 8-hole per inch wove wire.
1032	3·0	$1\frac{1}{8}$ pres.	$2\frac{1}{8}$	3186	3395	53 wet, 62 dry	30·2	85·6	3·37	1·14	33·9	60·4	94·9	·82	·89	One piece of 30-hole per inch wove wire.
1173	5·0	$2\frac{1}{2}$	$3\frac{1}{2}$	2875	3064	64 wet, 76 dry	30·2	100·7	3 63	1·71	47·0	77·4	75·4	·58	·63	One piece of 54-hole per inch wove wire.
1291	3·7	$3\frac{7}{8}$	$4\frac{3}{4}$	2534	2700	44 wet, 54 dry	30·2	110·8	3·44	2·04	59·4	87·0	60·4	·43	·47	One sheet of perforated zinc. Best experiment
1485	2·7	$5\frac{5}{8}$	6	1888	2012	63 wet, 78 dry	30·1	127·4	3·22	1·93	59·9	83·2	39·1	·29	·31	Two sheets of perforated zinc, superposed.
1553	2·7	$5\frac{3}{4}$	$5\frac{7}{8}$	1417	1510	54 wet, 65 dry	30·2	133·3	2·33	1·41	60·6	74·4	28·1	·22	·24	Three sheets of perforated zinc, superposed.
1525	1·3	$5\frac{1}{4}$	$5\frac{1}{2}$	726	773	46 wet, 52 dry	30·2	130·9	1·36	·68	49·9	72·3	14·6	·12	·13	Four sheets of perforated zinc, superposed.
1527	5·0	$3\frac{13}{16}$	$3\frac{7}{8}$	..	..	....	30·1	131·1	1·41	..	..	50·0	..	..	..	Entirely blocked.

Radius of vanes 9·8 in. Area of the end of the pipe at Z less that of the wires, 1·066 square feet.

results of the experiments on the different fans with maximum and minimum equivalent orifices, together with particulars as to speeds, &c., observed in the tests.

Experimental Apparatus.—A conical piece of pipe X Y, fig. 42, fitting the outlet of each fan, was bolted, as shown, to the fan under test on one side, and to a wrought-iron pipe,

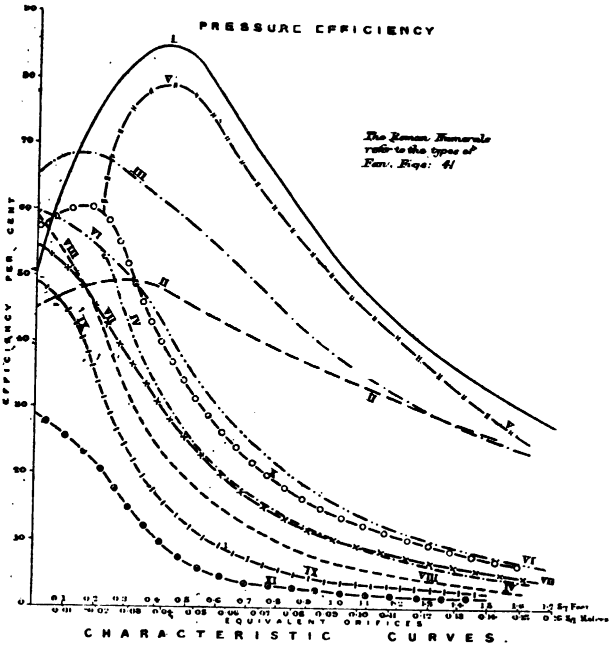


FIG. 44.

$14\frac{1}{32}$ in. in diameter, at the other. This latter was used for all experiments. Each fan was driven from a small engine by a strap from a rigger fixed on the crank shaft. Two assistants started the counters by signal at the same instant, and at the end of the experiment they were similarly thrown out of gear. By this means the speeds and slip of the strap

TABLE III.—SUMMARY OF THE BEST EXPERIMENT ON EACH OF THE ELEVEN TYPES OF FAN.

No. of fan.	Type of fan.	Diameter over vanes in inches.	Revolutions of fan per minute.	Static pressure at outlet of fan A in inches of water.	Dynamic pressure before baffle B inches of water.	Cubic feet of air per minute.	I.H.P. of fan only.	Theoretical H.P.	Mechanical efficiency per cent.	Pressure efficiency per cent.	Volumetric efficiency per cent.	Equivalent orifice in square feet.	Temperature of air at end of pipe F.	Barometric pressure in mercury.	Direction of vanes.
I.	20 vanes, W.I., 1⅛ in. wide at tip......	19⅝	1291	3⅞	4¾	2700	3·44	2·04	59·40	87·00	60·38	·44	44 wet, 54 dry	30·2	Concave
II.	12 vanes, C.I., 2½ in. wide at tip......	25½	1354	4½	4⅞	1905	3·31	1·47	44·49	48·68	18·65	·33	75 wet, 77½ dry	30·08	Concave
III.	6 vanes, W.I., 7 in. wide at tip......	23¼	1248	4¼	$4\frac{7}{16}$	1657	1·93	1·18	60·92	63·20	23·34	·30	74 wet, 82½ dry	30·36	Radial
IV.	8 vanes, W.I., and 8 half vanes, 4⅛ in. wide at tip......	23⅝	1014	$4\frac{3}{16}$	$5\frac{3}{16}$	1261	3·16	1·05	33·13	52·65	20·75	·21	55 wet, 68 dry	29·8	Concave
V.	24 vanes, W.I., 3½ in. wide at tip......	15¾	1500	3⅜	$3\frac{11}{16}$	2286	2·4	1·35	56·35	79·80	85·90	·45	88 wet, 91 dry	30 28	Concave
VI.	6 vanes, W.I.	20	1589	3⅞	4¼	1636	2·39	1·11	46·59	50·49	28·33	·30	69 wet, 74 dry	29·70	Convex
VII.	6 vanes, W.I., 3¼ in. wide at tip......	$24\frac{15}{16}$	1355	$3\frac{7}{16}$	3¾	1526	2·47	·91	36 95	39·02	15·93	·30	61 wet, 69 dry	30·20	Convex
VIII.	6 vanes, W.I., 1⅝ in. wide at tip......	24	1352	..	$3\frac{9}{16}$	1163	1·66	·66	39·57	39·59	13·82	·24	56 wet, 61 dry	30·50	Convex
IX.	4 vanes, C.I., 2⅛ in. wide at tip......	24⅝	1669	5	$5\frac{11}{16}$	1074	2 11	·97	46·10	38·88	9·48	·16	55 wet, 63 dry	30·34	Concave
X.	18 vanes, 12 small, 6 large, $1\frac{3}{16}$ in. wide at tip......	23⅝	1747	9⅛	8⅞	1280	3·22	1·82	56·57	59·25	12·22	·17	57 wet, 61 dry	30·00	Straight but not radial
XI.	10 vanes, C.I., ¾ in. wide at tip......	15¾	2097	$1\frac{5}{16}$	$1\frac{5}{16}$	913	·66	·19	29·34	14·31	25·17	·31	57 wet, 63 dry	30·52	Concave

were ascertained. At A and B two pipes of $\frac{1}{4}$ in. diameter were fixed to the air pipe Y Z, and to these were attached two U water gauges, by which the static pressures of air were observed. The dynamic pressures were obtained by means of a dial gauge of special design. The circular pieces of wove wire or perforated zinc were fixed inside the pipe at C,

MECHANICAL EFFICIENCY.

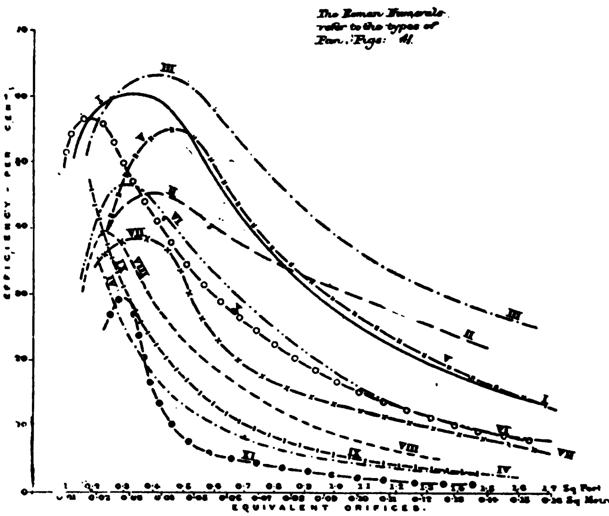


FIG. 45.

causing the air to be uniformly baffled, diminishing the flow through the fan, but increasing the pressures as the areas were successively reduced. These pieces of wire or zinc were changed for each experiment. The free end of the pipe at Z delivered the air from the fan into the atmosphere. The Pitot tube system was adopted for the measurement of the air as being quite as correct as, and more convenient

TABLE IV.—VARIATIONS BETWEEN THE RESULTS OF EXPERIMENTS WITH MAXIMUM AND MINIMUM EQUIVALENT ORIFICES ON EACH FAN.

No. of Fan.	Kind of baffle in pipe.	Revolutions per minute.	Maximum and minimum equivalent orifices.	Static pressure at A.	Dynamic pressure at B.	Velocity of air at end of pipe in feet per minute.	Quantity of air in cubic feet per minute.	Slip of strap per cent.	Mechanical efficiency per cent.	Pressure efficiency per cent.	Volumetric efficiency per cent.
I.	3-hole wire.	857	1·52	·5 vac.	·7	3,319	3.536	1·2	17·0	34·0	119·0
	4 sheets of zinc.	1 525	·13	5·75 press.	6·0	726	773	5·0	50·0	72·0	15·0
II.	3-hole wire.	876	1·30	·30 press.	1·2	3,523	3,755	4·4	25 0	29·0	57·0
	4 sheets of zinc.	1,499	·13	5·75 press.	6·0	1,063	1,134	2·8	45·0	49·0	10 0
III.	3-hole wire.	980	1·43	·16 press.	1·0	3,827	4,077	5·5	32·0	27·0	73·0
	4 sheets of zinc.	1,088	·17	5·10 press.	5·2	795	847	·5	61·0	68·0	14·0
IV.	3-hole wire.	706	1·44	1·00 vac.	·2	1,842	1,962	5·0	3·0	5·0	46·0
	3 sheets of zinc.	1,014	·21	4·25 press.	5·0	1,183	1,262	·7	33·0	53·0	21·0
V.	3-hole wire.	1,074	1·26	·05 press.	·8	3,009	3,206	2·7	22·0	38·0	167·0
	4 sheets of zinc.	1,056	·22	3·30 press.	3·7	638	678	·5	56·0	80·0	36·0
VI.	3-hole wire.	1,289	1·42	0 press.	·5	2,409	2,566	8·5	10·0	8·5	55·0
	5 sheets of zinc.	1,500	·16	4·40 press.	4·7	785	837	5·5	46·0	55·0	15·0
VII.	3-hole wire.	1,276	1·49	0 press.	·5	2,522	2,687	3·8	9·0	5·5	30·0
	4 sheets of zinc.	1,185	·21	4·00 press.	4·1	945	1,007	4·9	37·0	45·5	12·0
VIII.	3-hole wire.	1,300	1·27	1·00 vac.	·4	1,856	1,977	2·6	6 0	4·0	24·0
	4 sheets of zinc.	1,186	·16	3·00 press.	3·5	715	761	5·2	49·0	47·0	0·0
IX.	3-hole wire.	1,184	1·23	·67 press.	·2	1,583	1,687	·5	4·0	2·6	17·6
	4 sheets of zinc.	1,669	·17	5·00 press.	5·6	1,008	1,076	3 0	46·0	39	9·5
X.	3-hole wire.	1,102	1·48	·04 press.	·5	2,663	2,840	7·0	10·5	8·0	40·0
	7 sheets of zinc.	2 099	·096	12¼ press.	12·3	815	867	10 0	56·6	61·0	7·0
XI.	3-hole wire.	1,565	1·45	0 press.	0	1,008	1,159	4·5	1·0	1·0	30·0
	3 sheets of zinc.	2,541	·24	2·67 press.	2·6	756	814	5·2	24·0	19·0	23·0

than, an anemometer. We think that the reason that the flow was correctly recorded was that the air was uniformly baffled over the whole surface of the section of the discharge pipe, and eddies, which are set up by sudden contraction at a single orifice, as was the case in the experiments made by Messrs. Heenan and Gilbert, were avoided. An important point was to determine the mean speed of the air in the pipe Y Z.

The Breslau experiments, made by the Prussian Mining Commission, in 1884, showed that anemometers give too high results when calibrated in the usual way in a circular path. The speeds of the air on issuing from the pipe into the atmosphere were measured at Z in the following way: A carefully-made brass Pitot tube, ¼ in. in internal diameter, was held in the hand, with its open end facing the currents of air at Z. The other end was connected by an indiarubber pipe to a special dial water gauge, fig. 42, on which the dynamic pressures due to varying velocities of air, at different positions in the pipe, were indicated. On this gauge a deflection of 33 millimetres represented a water pressure of 1 millimetre. At the free end of the pipe at Z a template of wire was fixed, dividing it into eight equal areas. At the centre of gravity of each of these areas the Pitot tube was supported by the wire template. The mean of these eight readings in each experiment was taken as the average pressure in millimetres of water. (As a measurement of the mean velocity it would have been more accurate to take the mean of the square roots of all the readings, as we have shown that

$$v = \sqrt{\frac{\rho g h}{6 D}}, \text{ i.e., } \propto \sqrt{h},$$

if h is measured in inches, and ρ and D are the densities of water and air.) The dynamic air pressure was also taken at a distance of two-thirds of the radius from the centre of the pipe. The Breslau experiments showed that this particular position gave the mean dynamic air pressure in the pipe due to the mean velocity, a result which was confirmed by Mr. Donkin's tests. The mean dynamic pressure at Z having been thus obtained, the mean velocity of the air was deduced from the formula (2) given in the list of formulæ used in the experiments. The mean velocity having been determined, the quantity of air in cubic feet per second was obtained by multiplying the velocity by the area of the pipe. The latter was gauged inside the end of the pipe Z, allowance being made for the wire template. The temperature of the air at Z was noted in each experiment with a wet and dry bulb thermometer, and the barometric pressure was also observed.

20. *Types of Fans Used in the above Experiments.*—Fan No. I.: This fan was made with twenty specially-shaped wrought-iron vanes curved in two directions, and revolving with the concave side to the outlet. The casing, of volute form, gradually increased in cross-sectional area towards the outlet, and there was only one inlet for air of special bell-mouthed form. The driving shaft was fixed in the centre of the inlet, with a cone on its end to guide the air to the revolving blades, and there was no bearing or obstruction to prevent the air from entering freely. The two brass bearings for the shaft, one with three thrust collars and one plain, measured $1\frac{3}{8}$ in. diameter by $5\frac{1}{4}$ in. long. This fan worked very quietly. In this case details of the ten experiments are given in Table II. Little attention was paid to ensuring any particular speed in each experiment, as it was found in the fans tested that, all other conditions being the same, the quantity of air delivered was proportional to the speed. The three efficiencies of this fan are all high. Two special experiments were made with a sheet of perforated zinc in the pipe—one with the vanes varnished and covered with coal dust to represent dirty vanes in a coal mine, and the other with vanes clean and bright. Corrected for speed, the results of the comparison showed that $10\frac{1}{2}$ per cent more air was delivered by the clean vanes, but the mechanical efficiency was about the same. The pressure efficiency was 11 per cent and the volumetric efficiency 6 per cent higher with clean than with dirty vanes. Two experiments were made, one with the large bell-mouthed inlet fixed in place as designed, and one with it removed. Corrected for speed, the result showed that 31 per cent more air was passed when the inlet was used, and the mechanical efficiency was 9 per cent higher. The pressure efficiency was 33 per cent, and the

volumetric 15 per cent higher. This proves the advantage of admitting the air without shock and in the right direction. In fig. 42a is given the variation of the pressures at the end of the pipe, taken with a Pitot tube connected with the dial water gauge, in an experiment on this fan. The pressure was taken in each of the eight equal areas. In this experiment, with two sheets of perforated zinc in the pipe, the mean of the eight readings gave a velocity of 1,888 ft. per minute (Table II.). The velocity obtained from the pressure at the centre of the pipe taken at the same time was 1,929 ft. per minute, and that taken from the mean of the pressures at points two-thirds of the radius from the centre was 1,896 ft. per minute.

Fan No. II. had twelve short curved vanes cast in one piece, revolving concave to the outlet, but some experiments were made with the vanes running in the opposite direction to ascertain the effect on the pressures and quantities of air. The outer cast-iron casing was of volute form gradually increasing in area towards the outlet. It was provided with two central air inlets. Two experiments were made with the vanes reversed and revolving convex to the outlet. The increase in the quantity of air delivered due to the vanes revolving concave to the outlet was found to be 11 per cent. The mechanical efficiency was 5 per cent less in the latter case, but in the pressure efficiency a gain of $5\frac{1}{2}$ per cent and in volumetric efficiency a gain of $2\frac{1}{2}$ per cent were effected.

Fan No. III. was the simplest tested, and had only six short, straight radial wrought-iron vanes with two inlets for air; the cast-iron casing was eccentric to the shaft, so that the cross-sectional area gradually increased towards the outlet. The fan worked quietly and the three efficiencies occupied a relatively good position.

Fan No. IV. was of a special duplex type, the air passing from an outer to an inner casing, and from thence to the outlet. The vanes were mounted on a centre plate of wrought iron, so that one-half of them were in the outer and the other half in the inner casing. There were eight large and eight small vanes on each side of the centre plate revolving concave to the outlet. The bearings were three in number, $1\frac{1}{8}$ in. diameter and 5 in. long, of white metal.

Fan No. V. was made with twenty-four short-curved wrought-iron vanes, intended to revolve with the concave side to the outlet. It had two bell-mouthed inlets to direct the entering air. In addition there were cones mounted on the shaft with the same object. The outer casing was of cast iron of volute form, the area increasing gradually to the outlet. The two bearings, arranged to allow for any deflection of the shaft, were $1\frac{1}{16}$ in. in diameter by $4\frac{5}{8}$ in. long, and were placed well away from the inlets. It will be seen that the three efficiencies in the best experiment are all relatively high. One experiment was made with the two external bell-mouthed inlets removed, the directions of the vanes remaining the same to test their effect. The gain was $3\frac{1}{2}$ per cent in the quantity of air delivered—due to these inlets—and gains of $4\frac{1}{4}$ per cent in mechanical efficiency and $5\frac{3}{4}$ per cent in pressure efficiency.

Fan No. VI. had six vanes, but their curvature was not continuous, but at about the middle they were set back and then continued to the full radius They were intended by the maker to revolve convex to the outlet. There were two inlets and two cast-iron bearings, $1\frac{1}{2}$ in. diameter by 8 in. long. This fan worked with a humming noise. In the eleven experiments the three efficiencies were not high, and were lower than in many of the other fans. An experiment made with the vanes revolving concave to the outlet gave a gain in discharge of 25 per cent. The mechanical efficiency was about the same, and the pressure efficiency 19 per cent higher.

Fan No. VII. is a type that has been used in this country for many years. It has six wrought-iron vanes revolving convex to the outlet. The vanes are considerably curved, and longer than in the other types. There are two air inlets, but the bearings are placed very near them, an arrangement which prevents the free entrance of the air. The spindle, $1\frac{5}{8}$ in. in diameter, revolved, in bearings 7 in. long, centrally with the inlets and the cast-iron outer casing. The air passage, between the ends of the revolving vanes and the casing, was not increased in area towards the outlet, as in the other fans.

Fan No. VIII. was somewhat similar to the last mentioned, and had six wrought-iron vanes of considerable curvature intended to run convex to the outlet. There were two inlets for air, and the centre of the shaft was placed centrally with these, but slightly eccentric to the outer casing. The spindle was supported in two cast-iron bearings, $1\frac{1}{2}$ in. diameter and 7 in. long, which were close to the inlets. The efficiencies were somewhat low. Other experiments were made with vanes running concave to the outlet, to ascertain the effect on the quantity of air delivered, power, etc. The mechanical efficiency was 11 per cent less, the pressure efficiency 7 per cent more, and the volumetric efficiency about the same. Other experiments proved that the static head inside the casing increased gradually round the circumference from $4\frac{7}{8}$ in. of water just above the outlet to $5\frac{3}{8}$ in. near the outlet (so that the volute was imperfectly designed).

Fan No. IX. is of a type used in considerable numbers on the Continent. Four vanes, an unusually small number, of thin cast iron are provided, and are intended to revolve concave to the outlet. There are two inlets for the air, placed centrally to the shaft and also to the outer casing, which was of cast iron. The two bearings, with white metal linings, were 1 in. in diameter by 5 in. long. These were situated close to the inlets. The fan was always noisy, especially at small orifices.

Fan No. X. was designed specially to give a great air pressure, and not with a view of delivering a large quantity. There were in all eighteen vanes, six of which were whole vanes, with twelve half vanes interposed. They were all cast in one piece and straight, but not set radially to the centre. There were two inlets centrally with the shaft, coned to guide the entering air. The boss of the revolving part was also curved to assist the entrance of the air. The outer casing was of cast iron, centrally with the shaft and of rectangular section, gradually increasing in area to the outlet. The bearings were $1\frac{3}{16}$ in. diameter and 6 in. long, placed well away from the air inlets. Thirteen experiments were made with a different number of baffles. It will be seen that this fan gave fairly good results compared with others, but it must be remembered that it was designed for pressure.

One experiment was made with the vanes running in a contrary direction to that shown in the figure. The results corrected for speed showed that the quantity of air delivered was 31 per cent more, the mechanical efficiency 4 per cent less, the pressure efficiency $1\frac{1}{2}$ per cent less, and the volumetric efficiency 3 per cent higher. (This last is difficult to understand, for if the discharge was 31 per cent greater, the volumetric efficiency, which varies as the discharge and inversely as the speed for a given fan at a given orifice, should also be 31 per cent more.) The fan worked quietly. An attempt was made to obtain the pressure and velocity of the air inside the casing, between it and the revolving blades, when running at 1,758 revolutions per minute, and with three sheets of perforated zinc superposed in the pipe. A Pitot tube was held against the current close to the edge of the revolving blades, and also as near the inside of the casing as possible. The pressure was found to be higher when close to the vanes. At D, No. X., fig. 41, the dynamic pressure when the tube was held close up to the vanes was $11\frac{1}{8}$ in. of water, corresponding with a velocity of 13,080 ft. per minute, and with the tube as near the outer casing as possible the pressure was $9\frac{15}{16}$ in. of water, or 12,500 ft. per minute. At E, the opposite point on the circumference, the dynamic pressure when the tube was held close to the vanes was $11\frac{7}{8}$ in. head of water, or 13,690 ft. per minute, and with the tube as near the outer casing as possible $11\frac{1}{2}$ in., or 13,480 ft. per minute. Thus it will be seen that the velocity of the air increased round the inside of the casing with the direction of rotation for the same speed of fan. (If we can judge from the figure, the increase of section round the casing was not proportional to the angle, so that the ratio of the section to the quantity of air passing through it gradually decreased towards the outlet, and this would account for the increase of velocity mentioned. The less velocity near the outside of the casing was probably due to skin friction.) The static pressure at A in the experimental apparatus, fig. 42, at the same time was $9\frac{1}{8}$ in. of water, and the quantity of air delivered was 1,291 cubic feet per minute.

Fan No. XI. was made with blades fixed on one side only of a disc, having ten cast-iron slightly-curved thick vanes revolving concave to the outlet. It had only one special inlet, which constituted its peculiarity. The air was allowed to enter, not parallel with the fan shaft, as in all the other fans tested, but at right angles to it. In this way a rotary motion was given to the air before it came in contact with the revolving vanes. The object of this arrangement was to minimise friction. The experiments, however, show that it was of little use. The outer casing was of cast iron of volute form. There were two cast-iron bearings, $1\frac{1}{4}$ in. diameter, about 8 in. long. Nine experiments were made with the usual baffles. It will be seen that the three efficiencies occupy a low position in the plotted results.

Mr. Donkin draws the following practical conclusions from the above tests. It seems that few English and Continental manufacturers make experiments to ascertain the quantity of air delivered, the pressure, and the power absorbed. Sufficient attention is not often given to the admission of air to the centre of the fan to reduce friction. The number and shape of the vanes and their direction of rotation seem often to have been guessed at, and not deduced from experiment. Between 20 and 25 vanes give the best results. The shape of the blades, their number, and the space between them and the outer casing exercise a considerable influence on the various efficiencies. The final inclination or angle of the vanes at their circumference has more effect on the pressure of the air, and less on the mechanical and volumetric efficiencies. The revolving portion of the fan should always be accurately balanced. Mr. Donkin recommends continuous lubrication at high speeds to reduce journal friction, and allowance should be made for the deflection of the spindle. The pulley should not be too small or narrow, so as to reduce the slip of the strap. The friction of the air inside the casing is often excessive, and care should be taken to allow its entrance and passage through the vanes and out of the fan with a minimum of skin friction. Changes of direction and shocks, which reduce the losses of head with the high velocities of air, should be avoided as much as possible.

LIST OF FORMULÆ USED IN THE EXPERIMENTS.

$$H = \frac{h}{w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

where H = metres of air, h = pressure in millimetres of water, w = weight of 1 cubic metre of air in kilograms at the atmospheric pressure and temperature.

$$V = 4\sqrt{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

where V = velocity of air in metres per second at the end of the pipe Z, and h = pressure in millimetres of water.

$$\left.\begin{array}{l}\text{Theoretical H.P.*}\\ \text{in chevaux-vapeur}\end{array}\right\} = \frac{Q \times w \times H}{75} \quad . \quad . \quad (3a)$$

where Q = quantity of air delivered in cubic metres per second.

$$\left.\begin{array}{l}\text{Mechanical}\\ \text{efficiency}\end{array}\right\} = \frac{\text{theoretical H.P.*}}{\text{H.P. required to drive fan shaft}} \quad . \quad (4a)$$

$$\text{Pressure efficiency} = \frac{g\,H}{c_1^2} \quad . \quad . \quad . \quad (5a)$$

where g = 9·81 metres per second, and c_1 = peripheral speed.

$$\text{Volumetric efficiency} = \frac{Q}{c_1 r_1^2} \quad . \quad . \quad . \quad (6a)$$

where r_1 = external radius in metres.

$$\text{Orifice in square metres} = \frac{Q}{\sqrt{g\,H}} \quad . \quad . \quad (7a)$$

$$\text{Equivalent orifice (in square metres)} = \frac{Q}{\cdot 65\sqrt{2g\,H}} \quad . \quad (8a)$$

$$= \text{Orifice} \times 1{\cdot}088.$$

Appended to this table was a short account of the experiments made by the Prussian Mining Commission in 1884 on the measurement of air in a pipe by different methods, from a large air-holder at the Breslau gasworks.

* The writer does not make it clear whether or no the H.P. required to drive the fan shaft is also in chevaux-vapeur. One cheval-vapeur = ·986 English H.P.

CHAPTER IX.

21. *Summary of the Report of the Prussian Mining Commission of 1884.*—A spare gas-holder was used for measuring the volume of the air. It was 85·3 ft. in diameter, contained 70,634 cubic feet, and served to check the other methods adopted by the commission. The tests were probably the best that have been published on the measurement of air by the following methods: (1) by anemometers; (2) by Pitot tubes; (3) through circular and square orifices. The practical questions the commission endeavoured to solve by using this holder and causing the air to pass through a pipe were the following:—

1. Do the formulæ generally used for standardising anemometers in a circular path in still air give correct results or not?

2. Can the instrument known as the Pitot tube be applied practically for measuring the speeds of air; and, if so, what formula should be used for calculating the speed and quantity of air?

3. May the fall in pressure between one side and the other of a thin orifice interposed in a pipe be used for calculating the quantity of air; and, if so, what formula should be applied?

4. What is the loss of head due to friction in regard to length and diameter of pipe used?

About eighty careful experiments were made, and the results and calculations appear to have been well checked. The cast-iron pipe was 14·3 in. in diameter and 33 ft. long. For stopping and starting the anemometers quickly and accurately an electrical arrangement was adopted. The vertical fall of the air-holder in several places was carefully determined electrically. The first series of experiments in 1884 were made with the air-holder at a water pressure of $2\frac{7}{8}$ in., but in the 1885 tests the holder was loaded and the pressure was increased to $4\frac{1}{2}$ in. of water. The density and temperature of the air were noted, and the experiments were made during the autumn to avoid the heating effect of the sun upon the wrought-iron holder. V gauges were fixed at different parts of the pipe. A Pitot tube was used for measuring the dynamic pressures of air, not only at the centre and at two-thirds of the radius distant from it, but also round the inner circumference of the pipe. The circular orifices used in these experiments measured 7·03 in. and 9·96 in. in diameter. The square orifice measured 6·26 in. along the side. The rectangular orifice was 9·17 in. by 4·45 in. The experimental coefficient determined for the circular orifice was ·64, and for the square and rectangular orifices ·61. Four Casella anemometers and one Robinson anemometer were tested. The Casella anemometers, previously tested in the usual way at the end of a radius bar and compared with direct measurement of air from the holder, showed variable errors, the excess ranging between 7 and 13 per cent. (In our opinion this is only a part of the error which is obtained by the use of the anemometer, due probably to the fact that the instrument when tested sets a mass of air in rotary motion, and therefore does not pass through so much air in its circular passage as it should do, and fewer revolutions apparently correspond to more than the quantity of air corresponding to the distance through which it has moved. Hence, when used to test a uniform flow, such as that from the gas-holder, an exaggeration was observed. But when used to test the extremely irregular flow of air from a fan, a still greater exaggeration is produced, we believe, and this error probably amounts to about 30 per cent.) Anemometer readings should therefore be accepted with caution. In the 14·3 in. cast-iron pipe a considerable difference in speed was found at different parts and in the same vertical plane. The centre gave the maximum speed and pressure and the inner circumference the minimum; the mean speed of the air was found to be at two-thirds of the radius from the centre of the pipe. With regard to the resistance to the movement of the air in the pipe used, the following are the conclusions deduced from these experiments and given in the report: (1) that the resistance of the air increases as the square of its speed in the pipe; (2) the resistance to the air in the pipe decreases as the diameter of the pipe to the power $\frac{11}{8}$; (3)

the resistance of air in the pipe increases as the density of the air to the power $\frac{2}{3}$. These conclusions apply to cast-iron pipes only.

To obtain the quantity of air by means of a Pitot tube the following formula is given :—

Velocity of air in metres per second

$$= 4{\cdot}265 \sqrt{\frac{\text{pressure in millimetres of water}}{\text{density of air}}}$$

when the temperature of the air is zero Centigrade. If it is altered for the warmer temperatures of air it becomes very nearly the same as

$$V = 4\sqrt{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

the formula used by Mr. Bryan Donkin.

22. *Summary of the Report of the Belgian Commission on Mine Fans.**—The object of this commission was not scientific, but practical. They decided to examine a limited number of fans from the following points of view: (1) Quantity discharged; (2) mechanical efficiency; (3) safety of working; (4) cost of construction and erection; (5) accessory advantages and disadvantages. The great number of types of fans compelled them to limit their experiments to the following: The Guibal, Ser, Capell, and Rateau, and four of each type were chosen, but unfortunately the equivalent orifices of the fans were not varied, the orifices being those of the mines themselves. The revolutions per minute were the normal speed, ten turns above that and ten turns below, so that three experiments were made on each fan, when revolutions of fan and engine, water gauge, discharge, horse power, and in certain cases temperature and barometric pressure, were measured. The manometric tube was either in the fan drift or close to the eye of the fan, and was either at right angles to the direction of the current, or turned to face it. The fan drift was usually divided at some point into a number of rectangles, and the anemometer was held in each of these by a man in such a position that he did not obstruct the flow of the air.

* Les Ventilateurs de Mines, Revue Universelle des Mines, vol. xx., 1892.

The first type of fan tested was the Guibal, which, although the earliest used, may be found ventilating many mines at the present day, both in this country and abroad. Its wheel is generally of considerable diameter, carrying a number of vanes, and enveloped over most of the circumference, allowing the air to escape by a single opening, regulated by a shutter to suit the orifice of the mine. The air enters the eye, and by its centrifugal action it reaches the circumference and passes out at the chimney. The vanes of Guibals are sometimes plane, and inclined in the opposite direction to that of rotation, but are usually curved

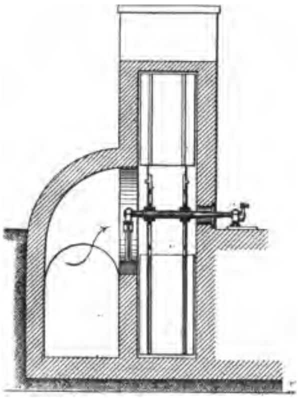


FIG. 46.

near the outer extremity until they become radial. The number of vanes lies between six and ten for sizes varying between 19 ft. and 40 ft. The breadth of wheel for these diameters lies between 4½ ft. and 10 ft. The chimney expands from the wheel to the mouth in order to reduce the velocity and increase the pressure of the air. Figs. 46, 47, 48, 49 show two examples of Guibals, the first of which is 39⅓ ft. in diameter, and the latter 19. The breadth of wheel of the first is 8¼ ft., and of the latter 6·4 ft. The Ser fan, figs. 50, 51, was designed in 1878 by Professor Ser, of the Ecole Centrale of Paris. The theory of this fan is

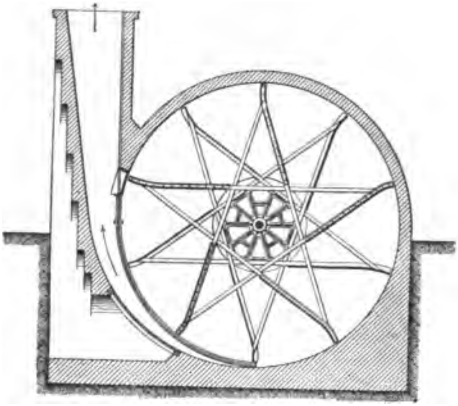


FIG. 47.

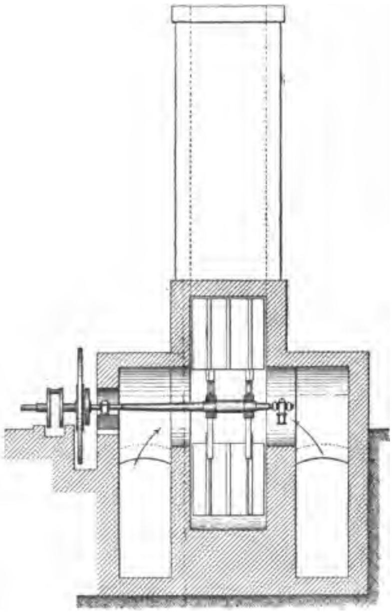


FIG. 48.

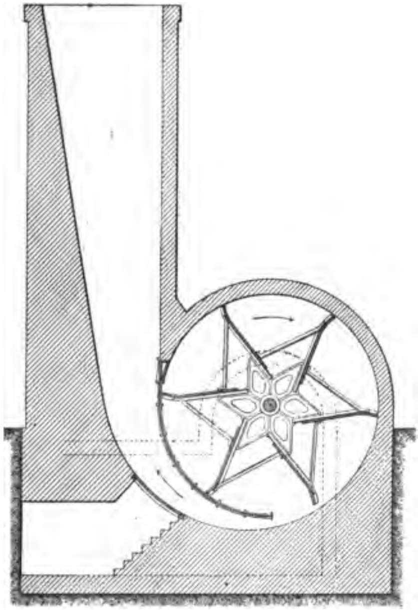


FIG. 49.

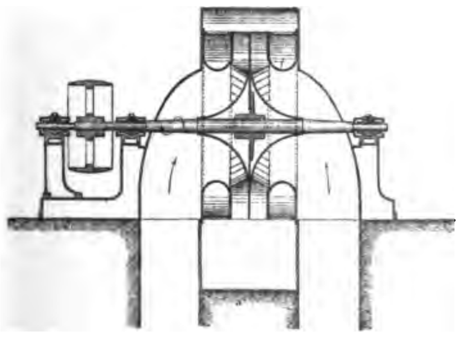


FIG. 50.

published in the "Mémoires de la Société des Ingenieurs Civils for 1878." It consists of a circular plate fixed to the shaft and carrying on each side 32 curved vanes, each of which forms part of a cylindrical surface whose generatrices are parallel to the shaft, and whose transverse section is circular. Their width is constant, and it is arranged that inflow shall take place without shock. The relative direction of outflow is at an angle of 45 deg. with the radius. Inflow takes place at both sides, and the fan is provided with a volute and expanding chimney. The volute is so designed

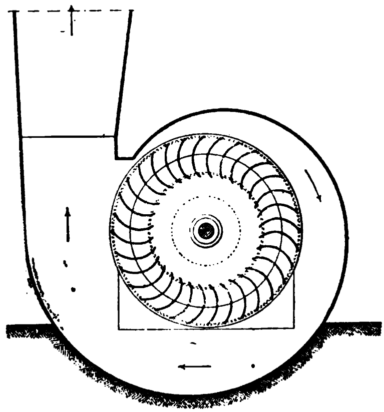


FIG. 51.

that the loss of energy at entrance from the circumference of the fan is a minimum, and the sides of the chimney are inclined at not more than 1 in 8 to avoid the loss due to sudden enlargement of passage. This type of fan is made from 4·6 ft. to 8·2 ft. The Capell fan, figs. 52, 53, is formed of two fans, one outside the other. The first consists of a drum of steel plate, closed on one side if there is a single eye. Its diameter is that of the eye. The cylindrical surface contains six openings, usually rectangular, spaced equally round the circumference, having an area less than the cylindrical surface of the drum, but not less than that of the eye. Six vanes of steel plates of cylindrical shape convex to the direction of outflow end at one of the sides of the openings in the drum, and within it. The second

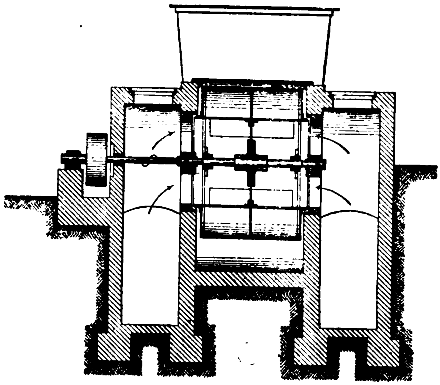


FIG. 52.

part of the wheel is larger than the first and is completely closed at the sides by two angular discs of steel plates; this part has six vanes curved backwards, and the casing is

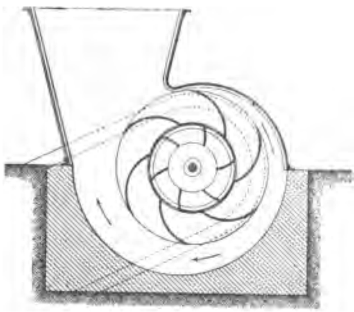


FIG. 53.

formed of a volute and a chimney of considerable taper. They are constructed up to 20 ft. for mine ventilation. The Rateau ventilator has already been described.

TABLE V.—EXPERIMENTS WITH GUIBALS.

No. of fan.	Revolutions per minute of engine.	Revolutions per minute of fan.	Tip-speed in feet per second. c_1	Engine H.P. in chevaux-vapeur. T.	Velocity of air in feet per second.	Cubic feet of air per second. Q	Water gauge in inches. h	Useful H.B. done on air in chevaux-vapeur. t	Equivalent orifice in square feet.	Mechanical efficiency per cent. $\frac{t}{T}$	Mean manometric efficiency per cent.	Mean mechanical efficiency per cent.	Volumetric efficiency per cent. $\frac{Q}{c_1 r_1^2}$
1	46	46	94·75	77·61	40·6	1960·	2·16	42·18	31·7	54·3	..	..	..
	56	56	101·5	140·49	45·2	2180·	3·31	71·82	28·6	51·1	53·9	52·5	..
	65	65	118·	205·49	53·0	2560	4·21	107·34	29·6	52·2	..	..	5·6
2	50	50	103·	59·94	20·2	837·5	2·87	23·94	11·7	39·9	..	..	..
	60	60	109·	89·74	23·8	987·5	4·10	40·21	11·6	44·8	60·7	42·4	..
	70	70	127·	152·10	27·2	1130·	5·79	64·99	11·2	42·7	..	..	2·3
3	40	64	98·75	58·49	14·6	961·	3·07	30·40	13·5	52·0	..	..	..
	47	75	115·5	89·55	16·2	1100·	4·01	46·78	13·1	52·2	66·7	50·4	..
	53	85	131·0	118·09	17·0	1155·	4·84	55·66	12·5	47·1	..	..	4·05
4	100	100	99·5	82·00	19·5	1565·	3·15	49·06	20·9	59·8	..	..	..
	106	106	105·	100·42	20·9	1670·	3·54	59·78	21·4	59·5	69·4	60·00	..
	120	120	119·	125·18	22·4	1800·	4·26	76·14	20·9	60·8	..	..	16·75

TABLE VI.—Experiments with Ser Fans.

No. of Fan.	Revolutions per minute of engine.	Revolutions per minute of fan.	Tip speed in feet per second. c_1	Engine H.P. in chevaux-vapeur. T	Velocity of air in feet per second.	Cubic feet of air per second. Q	Water gauge in inches. h	Useful H.P. done on air in chevaux-vapeur. t	Equivalent orifice in square feet.	Mechanical efficiency per cent. $\frac{t}{T}$	Mean manometric efficiency per cent.	Mean mechanical efficiency per cent.	Reduced equivalent orifices. $\frac{Q}{\cdot 65\sqrt{2gH} \times r_1^2}$	Volumetric efficiency per cent. $\frac{Q}{c_1 r_1^2}$
1	65	336	80·5	10·65	8·91	304·5	1·45	4·27	5·81	40·1	..	..	..	..
	75	391	93·6	16·86	10·64	364·0	1·85	6·47	6·12	38·4	47·7	40·5	..	..
	85	435	115·2	22·13	12·90	441·5	2·24	9·52	10·22	43·0	..	..	·6300	35·7
2	63	302	83·0	25·43	10·40	689·	1·93	12·78	11·39	50·3	..	..	..	..
	73	345	94·7	37·63	12·64	840·	2·68	21·60	11·81	57·4	64·3	54·6	..	..
	80	377	103·2	46·96	13·45	894·	3·07	26·37	11·70	56·2	..	..	1·704	126·
3	57	302	103·5	45·68	{23·7 / 5·15	1260·	1·61	19·60	22·9	42·9	..	..	..	..
	67	365	121·5	75·70	{27·2 / 5·95	1455·	2·36	33·16	21·7	43·8	34·5	44·9	..	..
	72	382	131·1	90·12	{31·5 / 6·85	1685·	2·68	43·35	23·6	48·1	..	..	4·48	244·
4	36	122	52·3	17·36	10·13	875·	·94	7·94	20·6	45·7	..	..	..	..
	46	154	66·0	28·55	12·60	1090·	1·38	14·43	21·4	50·5	73·4	48·0	..	..
	56	186	79·6	60·71	16·00	1385	2·09	29·08	22·1	47·9	..	..	1·312	103·5

TABLE VII.—EXPERIMENTS WITH CAPELL FANS.

No. of fan.	Revolutions per minute of engine.	Revolutions per minute of fan.	Tip-speed in feet per second. c_1	Engine H.P. in chevaux-vapeur. T.	Velocity of air in feet per second.	Cubic feet of air per second. Q	Water gauge in inches. h	Useful H.P. done on air in chevaux-vapeur. t	Equivalent orifice in square feet.	Mechanical efficiency per cent. $\frac{t}{T}$	Mean manometric efficiency per cent.	Mean mechanical efficiency per cent.	Volumetric efficiency per cent. $\frac{Q.}{c_1 r_1^2}$
1	46	184	118·	64·42	29·5	1198·	3·39	39·05	15·00	60·6	..	..	..
	56	224	144·	95·08	25·4	1485·	4·33	61·83	16·50	65·0	49·1	63·4	..
	66	264	170·	150·14	28·3	1656·	6·11	97·14	15·35	64·7	..	..	25·8
2	40	160	69·7	6·69	11·7	469·	1·06	4·69	10·2	70·1	..	..	..
	50	200	85·7	14·84	15·7	612·	1·89	10·97	10·3	73·9	53·4	69·8	..
	60	240	103·	21·06	17·3	676·	2·52	16·39	9·4	65·6	..	..	39·1
3	36	160	99·	30·72	14·7	676·	2·32	15·12	10·2	49·2	..	..	..
	40	178	110	41·11	16 6	767·	2·83	20·88	10·4	50·8	50·9	48·2	..
	44	196	121·	53·41	17·3	800·	3·11·	23·86	10 2	44·7	..	..	19·0

TABLE VIII.—EXPERIMENTS WITH RATEAU FANS.

No. of fan.	Revolutions per minute of engine.	Revolutions per minute of fan.	Tip speed in feet per second. c_1	Engine H.P. in chevaux-vapeur. T	Velocity of air in feet per second.	Cubic feet of air per second. Q	Water gauge in inches. h	Useful H.P. done on air in chevaux-vapeur. t	Equivalent orifice in square feet.	Mechanical efficiency per cent. $\frac{t}{T}$	Mean manometric efficiency per cent.	Mean mechanical efficiency per cent.	Reduced orifices. $\frac{Q}{\cdot65\sqrt{2g}\,H\,r_1^2}$	Volumetric efficiency per cent. $\frac{Q}{c_1\,r_1^2}$
1	100	177	60·6	19·83	26·4	815	1·22	9·54	16·95	48·1	..	..	..	..
	120	216	74·1	33·27	28·4	875	1·81	15·22	14·92	45·7	67·3	47·3	..	..
	125	255	87·6	6·24	31·3	965	1·89	17·52	16·41	48·3	..	..	1·53	102·
2	110	237	81·25	24·15	21·5	795	2·64	20·16	11·3	83·9	..	..	..	..
	120	264	90·50	31·15	22·5	832	3·19	25·48	10·7	81·8	87·5	82·7	..	..
	130	285	97·60	40·94	25·4	942	3·74	33·81	11·15	82 6	..	..	1·04	89·8
3	35	145	69·60	31·22	17·0	645	1·77	10·92	11·15	35·	..	..	..	..
	45	186	89·50	50·64	20·4	775	2·84	21·12	10·65	41·7	80·2	41·2	..	..
	55	228	109·40	86·76	26·7	990	4·29	40·80	11·05	47·0	..	..	·525	43·
4	80	115	53·4	16·28	14·7	897	1·30	11·19	18·05	68·7	..	..	..	..
	101½	150	72·1	32·27	18·6	1132	2·05	22·33	18·26	69·2	90·7	71·8	..	..
	114	170	81·7	41·69	20·5	1234	2·68	32·29	17·63	77·5	..	..	·830	73·2

TABLE IX.—Dimensions, Cost,

No. of fan	1	2	3	4	1	2
Type of fan	Guibal	Guibal	Guibal	Guibal	Ser.	Ser.
Date of installation	Jan. '77	Aug. '82	Sep. '86	Apl. '91	June '84	July '87
Period of service to Jan. 1, '92	15 yrs.	9 yrs. 4 mths.	5 yrs. 4 mths.	9 mths.	7 yrs. 6 mths.	4 yrs. 5 mths.
Total cost of plant in pounds.	1,240	1,200	1,236	1,120	1,280	820
Cost of fan alone in pounds	310	256	270	180	480	300
Diameter of wheel in feet	39·3	39·3	29·5	19	6·56	5·25
Breadth of wheel in feet	8·2	8·2	6·89	6·39	1·18	·917
Diameter of eye	13·1	13·1	9·82	6·88 2 eyes	3·84	3·11
Height of chimney above centre of shaft	35·7	32	22·9	34·4	29·5	19·65
Length of mouth of chimney.	8·8	11·2	7·7	10·65	7·87	5·25
Breadth of mouth of chimney.	8·2	8·5	6 9	6·56	7·71	5·25
Diameter of shaft in inches	12·5	10·2	11·0	11·4 and 6·3	6·7	5·1
Length of shaft between bearings in feet	14·5	12·5	12·3	15·5	9·7	6·46
Pulley diameter in feet	None	None	8·2	None.	3·28	2·13
Weight of moving parts in lbs.	44,000	39,600	20,650	14,050	1,192	..
Number of steam cylinders	1	1	1	1	1	1
Diameter of piston in ins.	26·7	24·4	24·4	16·5	19·6	13·8
Stroke in inches	33·4	39·4	33·4	29·5	27·5	29·5
Diameter of driving pulley in feet	None	None	13·1	None.	17·3	10·5
Distance between shaft centres in feet	None	None	26·2	None.	24·6	23
Dimensions of engine house in feet	42·6 × 13·1	..	55·7 × 16·4	10·6 × 32·8	26·2 × 23	26·2 × 23

AND PERIOD OF SERVICE OF FANS.

3	4	1	2	3	1	2	3	4
Ser.	Ser.	Capell	Capell	Capell	Rateau	Rateau	Rateau	Rateau
June '88	Jan. '91	Mar. '89	July '89	July '91	Oct. '90	Mar.' 91	July' 91	Aug. '91
3 yrs. 6 mths.	1 yr.	2 yrs. 10 mths.	2 yrs. 6 mths.	6 mths.	15 mths.	10 mths.	6 mths.	5 mths.
720	1,400	1,440	700	1,380	1,000	1,000	1,200	1,200
240	414	360	210	300	420	380	480	560
4·59	8·2	12·3	8·2	11·8	6·56	6·56	9·17	9·17
·786	1·47	6·56	5·9	5·25	·525	·525	·754	·745
2·79	4·91	6·87 2 eyes	4·59 1 eye	7·2 2 eyes	3·9	·9	5·5	5·5
22·6	36·9	12·45	7·2	14·25	..	13·1	18·3	24·2
5·15	7·87	10·5	9·2	11·9	Blowing fan.	10·65	8·45	19·65
5·15	7·71	8·2	7·87	7 2	..	10·65	7·18	19·65
4·32	7·46	10·4 & 7·86	7·86	7·86 and 7·06	4·3	4·3	6·7	5·3
5·85	9·25	21·5	18	18	4 9	4·9	4·5	6·5
1·64	3·93	3·6	2·46	2·95	3·28	3·28	4·9	4·9
..	1,480	26,400	15,400	22,000	3,520	3,520	8,460	8,460
1	2 tandem	2 cranks at right angles	1	1	1	1	1	1
11·8	15·1 and 19·7	20·5	15·7	17·7	12·8	14·9	29·5	14·9
25·6	31 5	31·5	23·6	29·5	19·6	23·6	47·2	23·6
8·85	13·1	14·4	9·8	13·1	5·9	7·3	20·3	7·2
19	19·7	32·8	29·5	24·6	16·4	23	41·8	16·4
26·2 × 19·8	26·2 × 19 6	23 × 29·5	36·1 × 12·1	26·2 × 22·9	26·2 × 13·1	32·8 × 27·8	39·3 × 19·6	27·8 × 13·1

In the experiments the following formulæ were used:—

Useful work in the chevaux-vapeur,* $t = \frac{Q\,h}{75}$ . . (1b)

where Q is the number of cubic metres per second and h the water gauge in millimetres of water.

Equivalent orifice o in square metres $= \cdot 38 \frac{Q}{\sqrt{h}}$. (2b)

Mechanical efficiency $= \frac{t}{T}$ . . . . (3b)

where T is the indicated horse power.

$$\text{Manometric efficiency} = \frac{g\,h}{u^2 w},$$

where u = the peripheral speed, and w the weight of a cubic metre of air, which is assumed as 1·2. Of course, g = 9·81.

In Tables V., VI., VII., VIII. will be found the results of the trials translated into English units.

Table IX. gives the principal dimensions of these ventilators, their cost and period of service. We have added volumetric efficiencies in Tables V. to VIII. The quantities of air are throughout far too great, as acknowledged by the experimenters on page 30 of the paper, but relatively to one another are of value. We believe the exaggeration amounts to about 30 per cent.

CHAPTER X.

22. *Experiments with Rateau Fans.*—The only experiments that we know of, made with a variable number of vanes, are those by M. Rateau, described in his work "Considerations sur les Turbo-Machines," in which a small fan ·82 ft. in diameter was tested for water gauge with 18, 24, and 30 vanes. The number of vanes that gave the best result was 18, but the difference was trifling. The highest water gauges were 53·4, 53, and 50·8 millimetres, and the

* Equal to ·986 of 1 horse power.

lowest 46·5, 48·9, and 44; but unfortunately no tests were made of mechanical efficiency at the same time. It is extremely probable that a fewer number of vanes would give a better mechanical efficiency, although the water gauge would probably fall. The experiments in Table X. were made with a Rateau fan having a diffuser, but no volute. Inflow took place from one side, and the wheel diameter was

TABLE X.—TEST OF A RATEAU FAN WITH DIFFUSER ONLY. DIAMETER, 4·59 FT.

No. of experiment.	Revolutions per minute.	Cubic feet of air per second. Q.	Work done on air in H.P. (French).	Work done by engine in H.P. (French).	Mechanical efficiency per cent.	Reduced orifices. $Q \div r_1^2 \sqrt{g} H$.	Manometric efficiency per cent.	Water gauge. In.
1	405	317	9·5	19	·50	·71	·75	3·15
2	405	454	13·9	25·3	·55	1·02	·76	3·19
3	405	549	16·0	26·6	·60	1·26	·72	3·13
4	510	690	31·3	45	·70	1·27	·69	4·72
5	411	641	18·2	37	·49	1·49	·64	2·95
6	518	802	35·0	55·8	·63	1·50	·63	4·52
7	414	682	17·1	42·4	·40	1·70	·57	2·6
8	481	791	27·0	61·9	·44	1·68	·57	3·54
9	405	752	15·6	48·2	·36	2·03	·43	2·16
10	458	850	23·1	65	·36	2·02	·49	2·84
11	402	827	14·1	43·6	·32	2·50	·43	1·77
12	425	872	16·8	51·9	·32	2·47	·43	2·01

4·59 ft. (fig. 54). The results obtained with it were, however, inferior to those with fans having diffuser and volute, and it was replaced by a better. The trials were very carefully made, as we see that, although 3 and 4, 5 and 6, 7 and 8, 9 and 10, 11 and 12 were made at different revolutions, but each pair with the same baffle, the reduced orifices, or 3 and 4, are very nearly the same, as also their manometric efficiencies; and the same may be said of each of the other pairs. The mechanical, which is the indicated, efficiency increases with the revolutions and power in the three first pairs, and is apparently the same in the last two. The increase is just what we might expect, as the efficiency of the fan alone is constant, and that of the engine increases

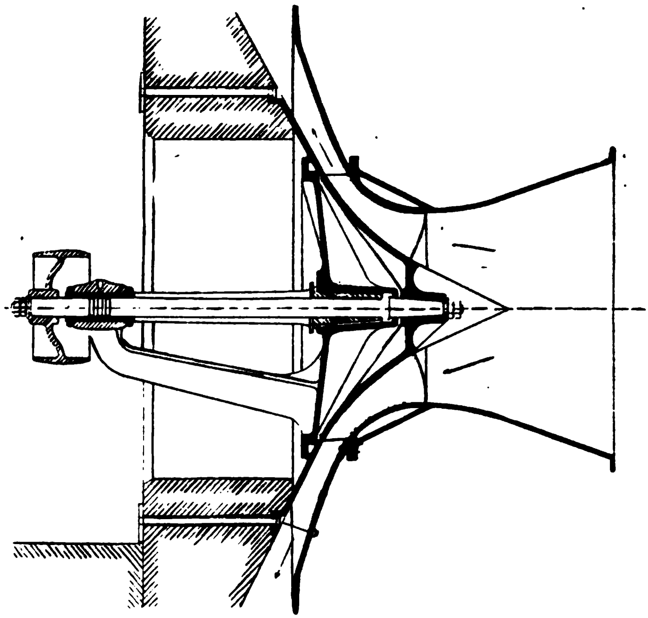


FIG. 54.

with the power, so that at a constant orifice the efficiency of the two should increase with the power.

Table XI.* shows the results of a series of experiments made with a Rateau fan of 4·59 ft. diameter, of type A, Table I., tested October 18th, 1891. Each experiment took three minutes. The water gauge was taken in an outlet

* Turbo-Machinrs, p. 166.

from the fan drift sheltered from the current. The discharge was measured at the top of the chimney, which was divided into 36 equal areas, by two assistants. Tests were also made to find the density of the air, so that H might be found from *h*. It will be seen in the table that the water gauge has been "corrected." This means that the gauge due to the velocity of discharge has been added, and that due to the

TABLE XI.—TEST OF A RATEAU FAN (TYPE A, TABLE I.) OF 4·59 FT. DIAMETER.

No. of test.	Revolutions per minute of fan.	Cubic feet of air per minute. Q.	Corrected water gauge in inches. *h*.	Equivalent head of air in feet. H.	Equivalent orifice. Square feet.	Reduced orifice. $\frac{Q}{r_1^2\sqrt{g\,H}}$.	Manometric efficiency per cent.
1	274	546	·99	71·4	12·69	2·19	53
2	359	614	2·05	146·8	9·99	1·70	64
3	380	635	2·38	170·5	9·45	1·63	67
4	385	592	2·82	201·5	8·16	1·41	77
5	372	519	3·11	222·7	6·76	1·17	91
6	263	324	1·66	118·7	5·80	1·00	96
6*a*	393	480	3·72	265·5	5·69	·99	98
7	425	418	4·49	321	4·83	·84	100
8	438	353	4·62	331	3·76	·65	97
9	444	254	4·31	310	2·79	·48	88
10	444	141	3·86	277	1·61	·28	78
11	340	42	1·93	138·5	·65	·11	67

velocity of inflow subtracted; while the latter was correct, the former was wrong, because the velocity of discharge, being of no value to anybody, ought not to be reckoned to the credit of the machine. If it were the custom to do so, of what use would a chimney be?

Table XII.* contains the results of 10 experiments with a Rateau fan of 6·56 ft. diameter at Villars. It is the same as

* Turbo-Machines, p. 170.

TABLE XII.—Test of a Rateau Fan (Type A, Table I.) of 6·56 Diameter.

No. of test.	Hole in baffle plate in square feet.	Revolutions per minute of engine.	Revolutions per minute of fan.	Observed water gauge in inches.	Corrected water gauge in inches. h.	Cubic feet of air per second. Section of chimney in 34·9 sq. ft. Q.	Mean velocity of air from the chimney in feet per sec.	Indicated H.P. in chevaux-vapeur.	Useful work done on air.	Depression in feet of air. H.	Equivalent orifice in square feet.	Reduced orifice. $\frac{Q}{r_1^2\sqrt{g\,H}}$.	Mechanical efficiency per cent.	Manometric efficiency per cent.
1	18·9	127	281	2·56	2·84	1243·	35·4	103·	34·	206·5	16·6	1·42	33	71
2	16·5	127	281	2·68	2·95	1232·	35·1	94·	35·	213·	16·2	1·38	37	74
3	14·2	130	290	3·07	3·35	1205·	34·4	91·	39·	242·5	14·9	1·27	43	79
4	11·8	138	306	3·74	3·97	1152·	33·1	96·	44·	288·	13·1	1·12	46	85
5	9·4	135	300	3·94	4·14	972·	27·9	74·	39·	302·	10·7	·92	53	91
6	7·1	147	326	5·00	5·07	825·	23·3	67·5	40·	371·	8·2	·70	60	95
7	4·72	147	326	4·92	4·96	582·	16·7	45·	28·	361·	5·9	·50	61	93
8	2·36	141	314	3·74	3·74	275·	7·86	23·	10·	275·	3·2	·27	43	75
9	0	126	279	1·97	1·97	45·9	1·3	15·	·7	144·	·75	·06	5	52
10	*	148	329	4·92	5·00	652·	18·7	53·	31·	371·	5·9	·55	59	94

* On the mine.

that tested by the Belgian Commission, and is No. 2, Table VIII. The tests were made in 1891, and show how greatly exaggerated were those in Table VIII.; we believe, however, that even these tests are slightly exaggerated. The discharge was measured in the same manner as in Table XI. by Casartelli anemometers, and the diagrams were taken with Crosby indicators, which could be trusted to give accurate results to 160 revolutions per minute. In tests 1 to 9 the fan was made to draw air from the mine and the atmosphere, and the orifice was altered by the resistance of a steel baffle plate placed at some distance from the fan. The last test was made on the mine in its natural condition, and as its results agree very closely with those obtained in the previous experiments, it was evident that the baffle plate was sufficiently far from the fan. The water gauge was taken in the fan drift by a Pitot tube, so that the effect of velocity in the fan drift was neutralised; but here again we find the value of h corrected, probably to take into account the energy rejected from the chimney, which was, of course, incorrect. We have, however, given these values.

Table XIII.* gives eight experiments with a Rateau fan of diameter 9·17 ft., on October 19th, 1891. The baffle plate was about 65½ ft. from the fan in the fan drift. The revolutions of the fan were taken by a counter, and those of the engine were calculated by dividing by 1·6. Experiments showed that there was extremely little slip, if any. The two following will suffice :—

Revolutions of fan	619	707
Revolutions of engine.........	387	440
Ratio	1·5995	1·5909

The water gauge was taken in the fan drift, and the end was not turned towards the current of air, so that a correction was necessary; but unfortunately, in this case also, we find the head due to the velocity of discharge added, which is incorrect, while that due to the velocity of inflow is subtracted, which is correct. The object of a chimney is to convert useless kinetic energy into useful pressure. The

† Turbo-Machines, p. 171.

TABLE XIII.—TESTS OF A RATEAU FAN 9·17 FT. DIAMETER, AT ALBI.

No. of Test.	Revolutions per minute of fan.	Revolutions per minute of engine.	Water gauge in inches (corrected). h	Equivalent head of air in feet. H	Mean velocity of discharge from the chimney.	Discharge in cubic feet per second. Q	Equivalent orifice in square feet.	Ratio of equivalent orifice to hole in baffle plate.	Useful work done on air, H.P.	Indicated horse power.	Mechanical efficiency per cent.	Manometric efficiency per cent.	Reduced orifice. $\frac{Q}{r_1^2\sqrt{gH}}$
1	231	144	4·1	298	6·62	493	5·59	1·26	19·4	..	..	78	·24
2	235	147	4·32	315	8·46	631	6·98	1·26	26·2	..	..	80	·30
3	227	142	4 56	334	11·96	895	9·56	1·21	39·3	60	66	90	·41
4	218	136	4·52	331	15·85	1181	12·80	1·25	51·5	78	66	97	·55
5	218	136	4·92	361	20·1	1491	15·15	1·22	70·7	90	79	106	·65
6	193	120	4·1	298	21·3	1591	18·05	1·21	62·7	98	64	112	·77
7	174	109	3·35	243	22·6	1690	21·25	1·23	54·4	94	58	112	·91
8	160	100	2·84	206	24·6	1830	25·1	1·20	50·4	90	56	112	1·08

section of the chimney was 74·6 square feet at the top, and the velocity of discharge at the maximum efficiency was only 20·1 ft. per second, corresponding to head in air of 6·3 ft., and as the total head was 361 ft., the difference is very trifling. The mechanical efficiency of 79 per cent is probably too high, and Professor Rateau suggests that it is due to the indicator; our own opinion is that it is due to the exaggeration of the discharge, and possibly of the water gauge. The manometric efficiency reaches the value of 112 per cent, and this suggests that the water gauge is exaggerated; this exaggeration is probably due to the nearness of the baffle plate. It will be noticed that the equivalent orifice is always greater than the opening in the baffle plate in the ratio of about 1·2 to 1. This is partly due to the passages behind the orifice, but also to the fact that the coefficient of contraction may be greater than ·65. It probably increases with the size of the orifice.

Table XIV. gives experiments on a fan of the same size at Montrambert, made on December 6th, 1891. The engine had a cylinder 14·95 in. diameter, and a stroke of 23·6. All the horse powers given with fans of this type are chevaux-vapeur, and consequently ·985 of 1 English horse power.

Table XV. contains experiments on a Rateau fan, 13 ft. 1½ in. diameter, at the Consolidation Mines, Westphalia. The fan was guaranteed to give 175,000 cubic feet of air per minute, with a water gauge of 6 in. and an equivalent orifice of 28 square feet, which corresponds to a reduced orifice of 0·6 very nearly—*i.e.*,

$$\frac{Q}{r_1^2 \sqrt{g \text{H}}} = \cdot 6.$$

In making this calculation we must remember that the equivalent orifice is 1·088 the orifice $Q \div \sqrt{g \text{H}}$.

23. *Experiments with a Centrifugal Pump at the Wallsend Slipway.**—These experiments were made by the author on March 13th, 1897, and January 15th, 1898. Two pumps are provided to empty the company's dry dock, but one only was used in the experiments. The diameters of

* From "The Theory of the Centrifugal Pump and Fan," by the Author. N.E.C. Institution of Engineers and Shipbuilders, vol. xiv.

TABLE XIV.—TESTS OF A RATEAU FAN 9·17 FT. DIAMETER, AT MONTRAMBERT.

No. of test.	Revolutions per minute of fan.	Water gauge in inches (corrected). h	Equivalent head of air in feet. H	Velocity of air at the chimney.	Discharge in cubic feet per second. Q	Equivalent orifice in square feet.	Useful work done on air in horse power.	Indicated horse power.	Mechanical efficiency per cent.	Reduced orifice. $\frac{Q}{r_1\sqrt[x]{gH}}$	Manometric efficiency per cent.
1	180	2·93	213	8·91	635	8·37	18	32	56	·36	92
2	192	3·87	280	15·25	1,085	12·25	40	..	..	·55	107
3	187	3·91	283	19·95	1,415	16·32	53	..	..	·71	114
4	148	2·53	183	20·25	1,435	20·50	55	..	..	·90	117
5	154	2·29	167	25·5	1,831	27·40	40·5	84	48	1·19	99
6	148	1·60	116	26·7	1,899	34·00	29·3	85	35	1·48	73
7	180	3·49	252	23·6	1,680	20·40	56	99	57	·89	110
8	192	3·73	271	24·5	1,735	20·40	62	110	57	·89	103
9	136	1·98	144	18·45	1,305	21·10	25	47	53	·92	104
10	86	·83	60·6	13·10	928	23·20	7·3	..	..	102	113

suction and discharge are 36 in., that of the wheel 66 in., while its internal breadth at outflow is $5\frac{3}{4}$ in.; the internal diameter is 39 in., and the vanes are radial at the inner circumference, and curve back in the arc of a circle until they become tangents to the outer circumference. In both experiments special care was taken in closing the gates to minimise leakage, which was not measured in the first experiment, but owing to some doubts expressed as to their accuracy careful measurements were made at the end of the

TABLE XV.—EXPERIMENTS ON A RATEAU FAN, $13\frac{1}{8}$ FT. DIAMETER, AT THE CONSOLIDATION MINES, WESTPHALIA.

No. of experiment.	Revolutions of engine.	Revolutions of fan.	Volume in cubic feet per minute.	Water gauge in inches.	Horse power done on air.	Horse power of engine.	Mechanical efficiency.	Manometric efficiency.	Equivalent orifice, square feet.	Reduced orifice. $\frac{Q}{r_1^2 \sqrt{g H}}$
1	47·	157·	137800	4·7	100·9	141·	69·	90·	24·5	·570
2	59·	196·6	166300	6·8	180·	245·	70·	86·	24·4	·566
3	25·	85·	80500	1·3	15·75	25·43	62·	85·	27·1	·630
4	40·	135·	138600	3·3	68·	106·3	64·	85·5	29·4	·682
5	47·5	158·	163200	4·5	107·	165·6	64·5	84·5	29·6	·687
6	57·5	193·	197100	6·7	196·	297·5	65·	85·	29·3	·680
7	67·5	227·	216400	9·3	298·	401·8	67·5	86·	27·4	·636

second. There was not the least necessity to do so, as the quantity even at the greatest head was quite negligible. The suction pipe is 15 ft. 6 in. long and 36 in. diameter, and the discharge pipe enlarges with a bend to 54 in. diameter at the junction with the discharge pipe of the second pump. The remainder of the discharge pipe is 95 ft. in all, 54 in. diameter, with a right-angle bend. We do not think that the introduction of these experiments needs any apology, as, except for the fact that in a centrifugal pump ϕ should not

TABLE XVA.—EXPERIMENTS WITH SEVERAL RATEAU FANS, $11\frac{1}{8}$ FT. DIAMETER.*

Name of mining company.	Equivalent orifice in square feet.	Reduced orifice. $\frac{Q}{r_1^2\sqrt{g}H}$	Diameter of engine cylinder in inches.	Stroke in inches.	Boiler pressure in lbs.	Revolutions of engine.	Revolutions of fan.	Water gauge in inches (uncorrected).	Cubic feet of air per minute.	Useful horse power of fan.	Horse power of engine.	Mechanical efficiency per cent.	Manometric efficiency per cent.	Tip speed of fan in feet per second.	Velocity of air at the place of measurement in feet per minute.
Dannenbaum	18·4	·545	19·7	31·6	73·5	76·	258	9·37	147,100	220·55	291·7	74·8	93·9	155·5	3020
Zollern	20·95	·62	23·6	39·4	58·7	68·	248	8·4	159,000	213·1	300·5	70·8	90·2	145·	2230
König Wilhelm	23·1	·685	23·6	39·4	..	68·	247	8·59	177,000	243·2	344·	70·7	94·7	143·5	2760
Franziska Tiefban..	23·4	·695	17·7	35·5	73·5	67·5	227	7·8	170,500	213·	287·1	74·1	98·	132·5	1650
Neu-Iserlohn	25·5	·755	16·75 / 25·6	31·5	..	77·5	231	7·16	178,000	204·65	369·8	66·05	89·7	134·5	3270

* From Messrs. Schuchtemann and Kremer's Catalogue, Dortmund.

be greater than 90 deg.,* the rules for designing pumps and fans are precisely the same, and deductions from experiments with the former apply to the latter. The weight of water was assumed as $62\frac{1}{2}$ lb. per cubic feet in the first experiment and 64 in the second. In the former the effect of the tide on the density of the water was forgotten, and in the latter a sample of the water in the dock whose density it was intended to test did not reach the Rutherford College owing to an accident, and the author desired to make every allowance for the pump whose efficiency was low; but it must be remembered that the efficiency takes into account the friction of the pipes, so that that of the pump alone is greater. The losses of head in passing through the pump are—

(1) At inflow,

$$\frac{(c_2 - u_2 \cot \theta)^2}{2g} = \frac{c_2^2}{2g}$$

since $\theta = 90$ deg.

(2) At entrance into the volute,

$$\frac{(w_1 - v_4)^2}{2g}$$

(3) Due to surface friction,

$$\frac{F. v_4^2}{2g}$$

and the value of $\frac{F}{2g}$ in the first experiment that gives the best agreement between theory and practice is ·07, and in the second is not much different to this. There was probably a little air in the pipe when the first experiment was made, but none in the second, as the steam ejector, which was not used in the first case, was employed, and a glass tube was fitted in the discharge pipe with a cock at its lower end, and this was kept open until shortly before the outer water level fell below the top of the discharge pipe. The pipe from suction to discharge formed a sort of siphon, and for the greater part of its length between pumps and discharge it was horizontal

* From "The Theory of the Centrifugal Pump and Fan," by the Author. N.E.C. Institution of Engineers and Shipbuilders, vol. xiv., p. 48.

TABLE XVI.—TRIAL OF A CENTRIFUGAL PUMP AT

1. Fall of water inside dock in feet....	0	1	2	3	4	5	6
2. Head outside dock..................	Ft. In. 19 8	Ft. In. 19 8	Ft. In. 19 8	Ft. In. 20 0	Ft. In. 20 0	Ft. In. 20 0	Ft. In. 20 0
3. Head inside dock at end of interval..	19 0	18 0	17 0	16 0	15 0	14 0	13 0
4. Boiler pressure	120	113	107½	105	110	110	105
5. Interval in minutes	..	7½	7½	7½	7½	7½	8
6. Mean head	..	1⅓	2⅓	3⅓	4⅓	5⅓	6⅓
7. I.H.P..............................	..	220·7	239·6	233·25	240·6	248	247·2
8. W.H.P.	..	12·12	22·1	34·1	46·1	53·9	59 75
9. F.H.P.	..	9·14	10·1	10·15	10·4	11·4	11·4
10. S.H.P.	..	211·56	229·5	223·1	230·2	236·6	235·8
11. Hydraulic efficiency, per cent, η	..	5·74	9·62	15·27	20·4	22·4	25·3
12. Calculated efficiency, per cent, ηc ..	..	6·15	9·7	14·15	17·9	21·1	24·9
13. $\eta - \eta c$	..	−·41	−0·8	·57	2·5	1·3	0·4
14. Revolutions per minute	..	125·5	136·5	137·5	140·3	144	144
15. Velocity of discharge from pump....	..	12·7	12·7	12·72	12·75	12·2	11 4
16. Orifice $\frac{Q}{\sqrt{g H}}$ square feet............	..	16·25	10·8	8·66	7·5	6·5	5·62
17. Time	9 45	9 52½	10 0	10 7½	10 15	10 22½	10 30½
18. Quantity discharged during interval in cubic feet	..	41,209	40,428	40,408	40,383	38,784	38,761
19. $\frac{g H}{c_1 w^1}$* × 100	..	9·5	10·9	17·85	21·7	23·7	25·6

* The angle of relative discharge from the fan is assumed to be at 26° 26′ from a tangent

THE WALLSEND SLIPWAY, MARCH 13TH, 1897.

	8	9	10	11	12	13	14	15	16	17	18	19
Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. in.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.
19 10	19 10	19 10	19 10	19 10	19 9	19 6	19 3	19 2	19 2	19 0	19 0	18 9
12 0	11 0	10 0	9 0	8 0	7 0	6 0	5 0	4 0	3 0	2 0	1 0	..
105	105	105	110	110	110	110	110	110	110	110	110	110
8¼	8¼	8	9¼	9½	9¾	9¾	10½	10½	10½	10½	10½	10½
$7\frac{5}{12}$	8⅜	9⅜	10⅜	11⅜	12·29	13⅜	13⅞	14¾	15 7	16·58	17·5	18·375
253·8	246·75	245·9	259·1	255	255	261·5	255·8	254·5	272·7	271·25	253·25	257·75
66·1	72	76	79·25	84·6	89·4	95	91·1	96·75	99·25	102·7	103·2	105·5
12·08	11·63	12·3	13·44	13·76	13·76	14·42	14·3	14·56	15·6	16	16	16
241·7	235·12	233·6	245·6	241·24	241·24	247·08	241·5	239·94	257·1	255·25	237·5	241·75
27·4	30·7	32·5	32·3	35·1	37·1	38·5	37·75	40·4	38·55	40·1	43 6	43 5
27·4	30·1	32·9	34·2	36 6	38·5	39·3	40·25	41·8	41·1	42·25	44·4	44·5
0	0·6	0·4	− 2	−1·5	−1·4	−0·8	−2·5	−1·4	−2·55	−2·25	−0·4	− 1
145·9	144·8	147·1	151·1	152	152	154·5	154·1	155	158·7	159·2	157·7	159·7
11·1	10·75	10·1	9·56	9·3	9·0	9·02	8·17	8·15	7·9	7·71	7·35	7·2
4·8	4·66	4·16	3·73	3·43	3·2	3·11	2·74	2·65	2·431	2·37	2·195	2·09
10 38¾	10 47	10 55¾	11 5	11 14½	11 24¼	11 34	11 44½	11·55	12 5½	12 16	12 26½	12 37
38,738	37,525	37,503	37,480	37,280	37,255	37,234	36,341	36,218	35,095	34,278	32,698	31,900
27·7	30·6	31·5	30·7	32·3	34·45	35·6	35·8	36·9	36·2	37·8	40·0	39·7

to the periphery of fan, this being the mean angle. The angle varies between 30° and 22° 53′.

TABLE XVII.—TRIAL OF A CENTRIFUGAL PUMP AT

1. Fall of water inside dock in feet	0	1	2	3	4	5	6
	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In	Ft. In.
2. Head outside dock...	21 0½	20 9	20 2	19 9	19 6	19 1	18 10
3. Head inside dock at end of interval..	19 0	18 0	17 0	16 10	15 0	14 0	13 0
4. Boiler pressure	110	110	110	110	110	110	110
5. Interval in minutes and seconds	..	11·45	11·45	11·0	10·30	9·30	9·15
6. Mean head in feet	..	2·395	2·957	3·457	4·125	4·791	5·457
7. I.H.P..............................	..	83·75	81·4	105·1	123·6	144·35	150·5
8. W.H.P.*..	..	16·3	19·7	24·66	30·7	37·9	44·4
9. F.H.P.	..	7·06	7·08	7·5	8·26	8·76	9·1
10. S.H.P.	..	76·69	74·32	97·5	115·34	135·59	141·4
11. Hydraulic efficiency, per cent, η....	..	21·3	26·5	25·3	26·7	28·0	31·5
12. Calculated efficiency, per cent, ηc. ..	..	22·5	26·1	26·4	27·3	27·9	29·55
13. $\eta - \eta c$	..	−1·2	+0·4	+1·1	−0·6	+0·1	+1·95
14. Revolutions per minute	..	97·1	98·3	103·5	113·3	120·4	124·8
15. Velocity of discharge from pump....	..	8·25	8·1	8·65	9·075	9·63	9·9
16. Orifice $\frac{Q}{\sqrt{g H}}$ in square feet...........	..	6·65	5·85	5·82	5·55	5·475	5·26
17. Time in hours and minutes	9 41½	9 53¼	10 5	10 16	10 26¼	10 36	10 45¼
18. Quantity discharged during interval in cubic feet †	..	41,209	40,428	40,408	40,383	38,784	38,761

* In the above the weight of water

† This does not include leakage, which at the end of the trial was found to be 30·6

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7	8	9	10	11	12	13	14	15	16	17	18	19
Ft. In. 18 4	Ft. In. 18 1	Ft. In. 17 8	Ft. In 17 5	Ft. In. 17 1	Ft. In. 16 8	Ft In. 16 3	Ft. In. 15 11	Ft. In. 15 6	Ft. In. 15 5	Ft. In. 15 2	Ft. In. 14 11	Ft. In. 14 9
12 0	11 0	10 0	9 0	8 0	7 0	6 0	5 0	4 0	3 0	2 0	1 0	..
110	110	110	110	110	110	110	110	110	110	110	110	110
9·15	9·0	9·0	9·0	8·30	8·30	8·15	8·45	8·40	8·20	8·0	7·40	7·20
6·083	6·707	7·375	8·041	8·75	9·375	9·9575	10·582	11·375	11 9575	12·792	13·546	14·33
176·0	188·0	188·25	217·0	254 9	262·6	270·3	276·0	277·3	274·1	288·0	289·2	285·0
49·9	54·1	59·6	65·0	74·25	79·6	87·25	85·25	92·4	98·0	105·5	117·5	120·5
9·4	9·72	10·0	10·4	11·74	13·13	13·76	14·0	14·92	15·1	15·35	15·6	16·0
166·6	178·28	178·25	206·6	243 16	249·47	256·54	262·0	262·38	259·0	272·65	273·6	269·0
30·0	30·3	33·5	31·5	30·5	31·9	34·0	32·6	35·1	37·9	38·8	43·0	44·8
30·6	32·0	33·2	33·6	29·9	33·9	34·8	35 6	37·3	38·5	39·4	41·1	42·0
−0·6	−1·7	+0·3	−2 1	+0·6	−2·0	−0·8	−3·0	−2·2	−0·6	−0·6	+1·9	+2·8
128·0	132·5	135·7	140·1	145·5	150·2	152 0	154·0	156·3	157·8	158·1	158 4	159·3
9·86	9·83	9·83	9·83	10·35	10·35	10·65	9·8	9·85	9·95	10·1	10·1	10·26
4·99	4·72	4·50	4·32	4·36	4·21	4·2	3·77	3·64	3·57	3·52	3·4	3·88
10 54½	11 3½	11 12½	11 21½	11 30	11 38½	11 46¾	11 55½	12 4⅛	12 12½	12 20½	12 28⅛	12 35½
38,738	37,525	37,503	37,480	37,280	37,255	37,234	36,341	36,218	35,095	34,278	32,698	31,900

per cubic feet is taken at 64 lb.
cubic feet per minute, and was probably less during the remainder of the trial.

and not much below high-water level. The discharge, however, always took place below water. Only one pump and engine were used in both cases. The friction of the engine was taken by running the engine unloaded at speeds varying between 122 and 185 revolutions. This is called F.H.P., and the horse power transferred to the pump shaft

$$\text{S.H.P.} = \text{I.H.P.} - \text{F.H.P.}$$

This is not exactly correct, because the friction of a loaded engine is a little more than that of one running light, but the error is not very great.

The following is the method of comparing the efficiencies obtained from experiment and calculation: In the first trial at the 14th foot we find I.H.P. = 255·8; W.H.P. = 91·1; revolutions per minute = 154·1 (these were taken by a counter read every minute); mean head, 13·875; this was measured by two posts outside and inside, giving heights in feet above the sill of the dock, and each interval commenced at a foot on the inner scale. On the outer scale inches were measured by a pole which had alternate inches at its end painted in black and white for a foot length, and this held against the post enabled very accurate readings to be taken, as the motion of the water was not more than 4 in. on March 13th and not more than 2 in. on January 15th. We mention these details, as we believe there have been no experiments on pumps made with greater care than these. The quantity of water discharged was 36,341 cubic feet during an interval of $9\frac{3}{4}$ minutes. This gives a velocity of discharge of 8·17 ft. per second from the pump, and as F.H.P. at 140·5 revolutions was 10·4 and at 160 revolutions was 16, by interpolation at 154·1 it was 14·3.

Hence $\quad \text{S.H.P.} = \text{I.H.P.} - \text{F.H.P.} = 241{\cdot}5$

$$\text{Pump efficiency} = \frac{91{\cdot}1}{241{\cdot}5} = {\cdot}3775.$$

This differs from the hydraulic efficiency only by the bearing friction, and neglecting this we may write—

$$w_1 = \frac{g\,\text{H}}{c_1\,\eta} = \frac{32 \times 13{\cdot}875}{44{\cdot}15 \times {\cdot}3775}$$
$$= 26{\cdot}6,$$

and the losses of head

$$= L = \frac{(w_1 - v_4)^2}{64} + \frac{c_2^2}{64} + \cdot 07\, v_4^2$$

$$= 5{\cdot}3 + 10{\cdot}7 + 4{\cdot}66$$

$$= 20{\cdot}66 \text{ ft.}$$

The efficiency is also obviously

$$= \frac{H}{H + L} = \frac{13{\cdot}875}{34{\cdot}535} = \cdot 4025.$$

The difference between that found by experiment and calculation is $\cdot 3775 - \cdot 4025 = - \cdot 025$.

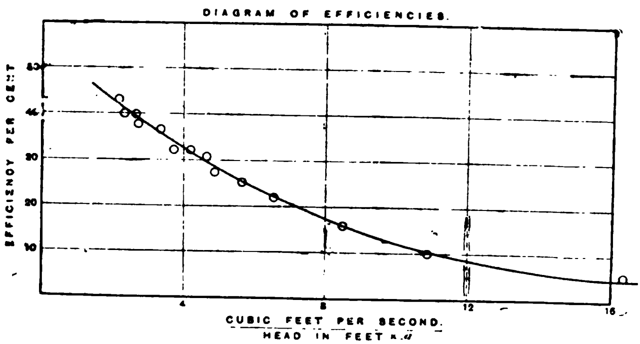


FIG. 55.

Fig. 55 is an efficiency diagram for the first experiment. The abscissæ are orifices in square feet—*i.e.*, cubic feet per second ÷ the square root of g H.

24. *Experiments with a Centrifugal Pump by the Hon. R. C. Parsons.**—In Table XVIII. we have selected a number of experiments made by Mr. Parsons with a centrifugal pump. After running the pump at various orifices an experiment was made to find the work required to revolve

* Proceedings of the Institution of Civil Engineers, vol. xlvii.

the pump at 410 revolutions (the mean of the number during the experiments), while the pump was not discharging. It was found that 45,000 ft.-lb. per minute were required, and this was deducted from the work done by the engine, and the work done by the pump being divided by this was called the corrected efficiency. This quantity is,

TABLE XVIII.—EXPERIMENTS BY MR. PARSONS WITH A CENTRIFUGAL PUMP.

No. of experiment.	Gallons per minute.	Lift in feet.	Foot-pounds raised per minute.	Foot-pounds indicated per minute.	Revolutions per minute.	Efficiency per cent.	Corrected efficiency per cent. η_3.	$\frac{g H}{c_1 w_1}$ η_1.	$\frac{H}{H+L}$ η_2.
1	1,012	14·67	148,461	208,438	392	49·74	58·57	57·5	57·9
4	1,280	14·7	188,160	343,754	398	54·74	62·99	58·7	60·1
6	1,431	14·75	211,073	374,954	400	56·20	63·95	60·0	62·1
8	1,568	14·75	231,280	404,737	403	57·01	64·29	61·2	61·25
10	1,695	14·75	251,987	419,790	405	60·17	67·18	62·1	65·0
11	1,753	14·8	259,450	435,630	406	59·42	66·39	63·25	65·9
12	1,012	17·4	176,088	370,458	424	47·53	54·06	56·2	53·75
15	1,280	17·3	221,440	417,214	428	53·08	59·51	58·7	60 0
17	1,431	17·4	248,994	447,552	431	53·63	61·86	60·0	60·75
19	1,568	17·4	272,832	471,552	433	57·86	63·95	61·2	63·7
21	1,695	17·6	298,310	486,050	435	61·37	67·64	62·6	64·7
22	1,753	17·6	308,528	494,210	436	62·43	68·68	63·5	64·9

however, more than the hydraulic efficiency, because the work required to drive a pump when not discharging is very much greater than the work expended in overcoming the friction of the fan shaft and the surface friction of the disc. The real hydraulic efficiency obviously is between this "corrected efficiency" and the ratio of the work done by the pump to that done by the engine which drove the belt. It will also be seen that the calculated efficiencies do lie between the latter and Mr. Parsons' corrected efficiencies, and are therefore close to the real efficiencies of the pump. These experiments help to prove that the mechanical efficiency of a pump is not far from

$$\eta_1 = \frac{g\,H}{c_1\,w_1}$$

or

$$\eta_2 = \frac{H}{H + L}$$

where L are the losses of head in feet, points that are dealt with in a subsequent article.

The method of calculating η_1 and η_2 is as follows: The dimensions of the pump are given in a paper on Centrifugal Pumps by Prof. Unwin (vol. liii., Proceedings of the Institution of Civil Engineers). The external radius of the fan is $r_1 = 9{\cdot}25$ in. $= 2\,r_2$, the internal radius. The breadths b_1 and b_2, at the external and internal radii, are both 5·75 in. There were eight vanes, and as their thickness is not given, they are assumed to be $\frac{1}{4}$ in. at their ends. The velocity v_4 in the volute is given by Prof. Unwin as $3\,u_1$, but as he evidently neglects the vanes, this must be modified. Assuming a coefficient of contraction of $\frac{9}{10}$ at discharge from the fan, which is also the custom in radial-flow turbines,* we get

$$v_4 = 2{\cdot}35\,u_1,\ u_2 = 1{\cdot}94\,u_1.$$

The angle ϕ made by the vane at the external radius with a tangent to the fan is 15 deg., and at the internal radius $\theta = 40$ deg., so that

$$\cot\phi = 3{\cdot}73,\ \cot\theta = 1{\cdot}91.$$

Let G = gallons per minute,

$$u_1 = \frac{G}{60 \times 6{\cdot}25\,(2\,\pi\,r_1\,b_1 - n\,b_1\,t_1\,\mathrm{cosec}\,\phi)\,K}$$

$$u_2 = \frac{G}{60 \times 6{\cdot}25\,(2\,\pi\,r_2\,b_2 - n\,b_2\,t_2\,\mathrm{cosec}\,\theta)\,K}$$

* Bodmer's Hydraulic Motors, Turbines, etc.

where $n = 8 =$ number of vanes

$$K = \frac{9}{10}$$

$$\therefore u_1 = G \times \cdot 001472.$$

Taking the first experiment as an example, $G = 1012$, $u_1 = 1\cdot 48$, $v_4 = 3\cdot 48$, $c_1 = 31\cdot 55$, $w_1 = c_1 - u_1 \cot \phi$; and assuming the relative angle of flow coincides with the angle of vane,

$$w_1 = 26\cdot 04,$$

$$100\,\eta_1 = \frac{g\,H}{c_1\,w_1} \times 100 = \frac{32\cdot 2 \times 14\cdot 67}{31\cdot 55 \times 26\cdot 04} = 57\cdot 5 \text{ per cent,}$$

$$\frac{(w_1 - v_4)^2}{64} = 7\cdot 8,\ \frac{u_1^2}{64} = \cdot 034;$$

then the loss of head at entry into the volute is

$$h_4 = \frac{(w_1 - v_4)^2}{64} + \frac{u_1^2}{64} = 7\cdot 834.$$

$$h_3 = \frac{(c_2 - u_2 \cot \theta)^2}{64} = 2\cdot 375.$$

The surface friction is $F\frac{v_4^2}{64}$, and the most suitable value of F is 2·5, so that

$$F\frac{v_4^2}{64} \doteq \cdot 4725.$$

As the tube by which the head at outflow from the pump was measured was a sort of Pitot tube turned to face the stream, the loss $\frac{v_4^2}{64}$ may be omitted; the efficiencies are, in fact, dynamic.

Hence, if L = total losses of head,

$$100\,\eta_2 = \frac{100\,H}{H + L} = \frac{14\cdot 67 \times 100}{14\cdot 67 + 10\cdot 681} = 57\cdot 9 \text{ per cent.}$$

We also find that the revolutions per minute R, H the head in feet, and G the gallons per minute are connected by the relation

$$R^2 + \cdot 02225\,G\,R - 9850\,H - \cdot 01285\,G^2 = 0,$$

which can readily be thrown into the form of equation (22). The following table gives the results of calculation from the above equation :—

No. of experiment.	Actual head.	Calculated head.	Difference.
1	14·67	15·1	− ·43
4	14·7	15·05	− ·35
6	14·75	14·86	− ·11
8	14·75	14·71	+ ·04
10	14·75	14·4	+ ·35
12	17·4	17·85	− ·45
15	17·3	17·66	− ·3
17	17·4	17·45	− ·05
19	17·4	17·35	+ ·05
21	17·6	17·05	+ ·55
22	17·6	17·16	+ ·46

25. *Experiments with Open-running Fans.*—In the Minutes of the Proceedings of the Institution of Mining Engineers, vol. xl., will be found a paper by Mr. Walton Brown, containing several experiments with a Waddle fan with an expanding rim, whose radial section is the same as the axial section of the large end of a trumpet. In other respects it is a simple open-running fan, with the vanes curved so as to be convex to the outlet. The following are the principal dimensions of the fan :—

Diameter to periphery of divergent outlet ...	36 ft. 4 in.
Diameter to the extremities of the blades ...	35 ft.
Diameter of inlet ring ...	13 ft. 6 in.
Width at outlet ...	1 ft. 1½ in.
Width at periphery of fan ...	2 ft. 2¾ in.
Capacity of fan ...	2,583 cubic feet.

The following experiments have been selected from those given :—

No. of experiment	1	2	4*	6	V.
Revolutions per minute of fan	50	57·2	59·73	37·2	67·47
Cubic feet of air per minute	140,158	169,953	231,306	105,777	174,008
Water gauge	2·070	2·410	2·671	1·082	3·274
H.P. in the air	45·68	64·5	97·33	18·51	89·68
I.H.P.	69·91	100·64	144·5	27·72	139·94
Efficiency	·653	·641	·673	·668	·641

* In this experiment the separation doors were open; the others were made on the mine.

In vol. xliii. of the same will be found several experiments with an open-running fan driven by a Corliss engine at Seghill Colliery. Its external diameter was 35 ft. 1 in., the inlet 12½ in. diameter on one side, and the inside width of the rim is 15½ in. It is made of extra thick steel plates, the front being $\frac{3}{16}$ in. thick, the back ¼ in., the back plates opposite to the inlet $\frac{7}{16}$ in., the vanes $\frac{3}{16}$ in., and it weighs about 19 tons. The fan shaft is of mild steel, 8 ft. 8 in. long between the bearings, 11½ in. diameter, and where the fan bosses are keyed on it is 12 in. diameter. The drift bearing, which carries two-thirds of the weight, is 10 in. diameter and 20 in. long; the bearing in the engine house is 8½ in. diameter and 14 in. long. The fan and shaft were made by Messrs. Thornewill and Warham. The engine was built by Messrs. Hick, Hargreaves, and Co., and has one horizontal high-pressure cylinder 16 in. diameter and 36 in. stroke of the Corliss type, working at a pressure of 100 lb. per square inch, but the steam pipes were 103 ft. long. The end of the water-gauge pipe in the drift was 10 ft. 4 in. from the centre of the fan shaft, with the end turned towards the fan (which would certainly tend to increase the gauge, but how much more so than if it were perpendicular to the current we are not prepared to say). The following are averages from several experiments, each average being the mean of a number of experiments which do not differ very much from it. The temperatures were between 57½ deg. and 60 deg. in the fan drift:—

Average of experiments	2 to 9	10 to 17	18*
Revolutions of fan per minute	39·806	60·44	60
Cubic feet of air per minute	87,910	135,742	247,520
Water gauge in drift	1·09	2·46	2·14
H.P. in the air	15·105	52·65	83·47
I.H.P.	29·612	96·114	153·67
Mechanical efficiency per cent	51·02	54·78	54·32

* Separation doors open.

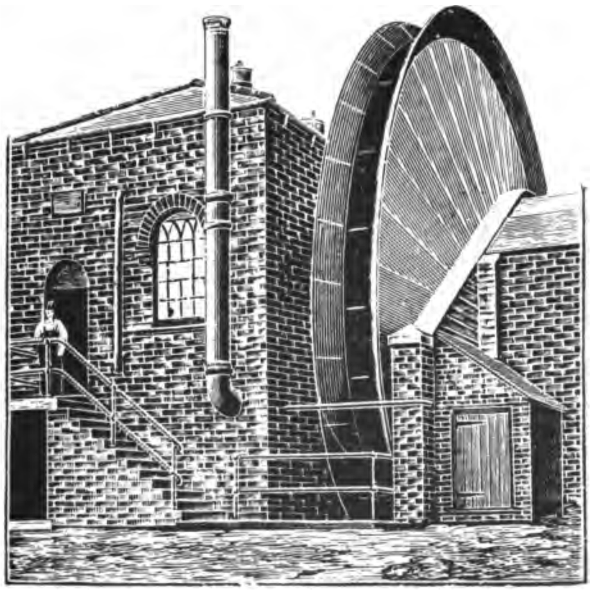


FIG. 56.

A view of the Waddle fan, described on page 127, is shown in fig. 56.

25A. *Experiments made by M. Lelong.**—M. Lelong has made a number of very interesting experiments in order to obtain data for the calculation of the dimensions of ventilators for warships. The first quantity requiring formulæ was the resistance of the circuit through which the fan discharged its air. This is made up of (1) surface friction, (2) changes of section, (3) changes of direction.

(1) *Surface Friction.*—The loss of head due to friction may be expressed by the formula (put in our own notation),

$$H_f = \frac{K\,l\,.\,\chi}{s}\,\frac{v^2}{2\,g}$$

where l is the length in feet, s the section in square feet, v the velocity in feet per second. K is a coefficient whose mean value is about 0·006, while χ is the perimeter of the section. Calling

$$\frac{2\,g\,H_f}{Q^2}$$

the resistance due to this, we get

$$\frac{2\,g\,H_f}{Q^2} = \frac{K\,.\,l\,\chi}{s^3} = \frac{\cdot 006\,l\,\chi}{s^3}.$$

(2) *Changes of Section.*—When a passage ends in a very large space, the kinetic energy of the current is completely lost, and the corresponding loss of head is

$$\frac{v^2}{2\,g},$$

while the resistance due to this

$$\frac{2\,g\,H_k}{Q^2} = \frac{1}{s^2}.$$

Inversely, if the current of air flows from a large space into a cylindrical pipe, we usually allow a coefficient of contraction, $\gamma = 0{\cdot}83$, for the vein entering the pipe, so that the loss of head here becomes

$$H_c = \left(\frac{1}{\gamma^2} - 1\right)\frac{v^2}{2\,g} = \left(\frac{1}{(\cdot 83)^2} - 1\right)\frac{v^2}{2\,g},$$

* Du Calcul des Ventilateurs, by M. Lelong, Ingenieur des Constructions Navales.

and the corresponding resistance

$$G = \frac{2\,g\,H_c}{Q^2} = \frac{0{\cdot}45}{s^2}.$$

In order to find the value of the coefficient that must be used for rectangular passages of large dimensions such as one finds on board ship, M. Lelong made several experiments. They were made with passages of two sizes. The section of the former was 1·31 ft. by 2·62 ft.; the smaller was obtained by dividing the first into two parts by a longitudinal partition. The passages were made of carefully planed wood, having a length of 9·84 ft. One end was connected with the atmosphere, the other was enclosed in a chamber into which a fan discharged. The static pressure in this chamber was given by a manometer; the discharge was given by an anemometer. The total resistance includes not only that at inflow, but also that due to friction and the loss of the kinetic energy at discharge; we have therefore deducted these two last, using a coefficient 0·004 for the coefficient of friction. The results obtained were given in the following table. They are left in metres, as the object of the experiments was to find a coefficient which is independent of the units :—

	Total pressure in the chamber in metres of air. H.	Discharge in cubic metres per second. Q.	Total resistance. $\frac{2gH}{Q^2}$.
First passage. Section 0·82 × 0·40 metres.	32	6·31	15·7
	28	5·90	15·7
	16	4·48	15·6
	8	3·19	15·5

Mean resistance.................... 15·6
Resistance due to friction......... 0·836
Resistance at outflow.............. 9·25
Remainder at inflow .... 5·51 = $\frac{0{\cdot}6}{s^2}$.

Value of corresponding coefficient of contraction, 0·79.

	H.		Q.		$\frac{2gH}{Q^2}$.
Second passage. 0·4 metres square.	8	......	1·36	......	84
	16	......	2·00	......	79·2
	28	......	2·63	......	79·2
	32	......	2·85	......	78

Mean resistance 80
Resistance due to friction .... 4·71
Resistance at outflow.................. 39·0
Remainder at inflow .... $36{\cdot}3 = \frac{0{\cdot}93}{s^2}$.

Value of corresponding coefficient of contraction = 0·72.

For the second passage the results given are the means between those obtained for each of the two passages.

By connecting the exit or entry of a pipe to the larger space by means of a cone, the loss of head is much reduced, and can become zero. It is negligible if the angle at the vertex of the cone does not exceed 30 deg. when the passage is to be reduced, and 7 deg. when the passage must increase.

(3) *Resistances due to Change of Direction.*—Changes of direction give rise to very variable losses of head, according to the different arrangements of the elbows. For sudden changes of direction, the loss is that given by Péclet, which agree very closely with the results obtained by other experimenters, and in particular by Weissbach.

$$H = \sin a \frac{v^2}{2g}$$

whence

$$G = \frac{\sin a}{s^2};$$

a being the angle made by one pipe with the prolongation of the other.

For bends the loss is much less. Weissbach gives the following formula :—

$$H = \beta \frac{v^2}{2g}$$

where

$$\beta = \left\{ 0{\cdot}131 + 1{\cdot}847 \left(\frac{d}{2\,\rho}\right)^{\frac{7}{2}} \right\}^*$$

for pipes of circular section, diameter d and mean radius ρ, and

$$\beta = \left\{ 0{\cdot}124 + 3{\cdot}104 \left(\frac{d}{2\,\rho}\right)^{\frac{7}{2}} \right\}^*$$

for rectangular sections having a height d. According to these formulæ the loss of head does not depend on the total angle of the bend, but on the ratio of d to ρ.

Several experiments were made by M. Lelong to see if Weissbach's formula could be practically applied to large rectangular-sectioned ventilation passages, such as are found in warships. The passages used were similar to those described above. At the ends of the bends it was necessary to add a length of passage of three metres, so as to obtain a uniform outflow of the air, which appeared to be extremely irregular after leaving the bends. The following table gives the results of experiments and compares the coefficients β thus obtained with those deduced from Weissbach's formula. Before the air entered the bend it had to pass through three metres of passage, and after leaving the bend through the same distance.

The values of β thus found by experiment are generally less than Weissbach's. The greatest discrepancies are to be found in experiments (4) and (8). These may be explained by the small angle of the bend of which Weissbach's formula as given by M. Lelong does not take account. The figures show, however, that the total resistance is very nearly independent of the discharge, and that the loss due to bends is less than that due to sharp corners, if we accept the formula

$$G = \frac{\sin \alpha}{s^2}$$

for these latter. In experiments (5) to (8) the passages were divided by vertical longitudinal partitions, and experiments (5A) and (7A) refer to those having the greater radii

* We have usually seen these formulæ multiplied by α, the fraction of two right angles of the bend, but even this modification does not bring about an agreement between Weissbach's formula and the results in the table.

No. of experiment.	Description of bend or bends.	Head in metres of air. H	Cubic metres of air per second. Q	Total resistance. $\frac{2gH}{Q^2}$	Mean resistance.	Resistance due to bends.	Experimental. β	Weissbach's. β
1	Right-angle bend	12·8 20	3·51 4·29	20·2 21·2	20·7 20·7	4 4	0·43 0·43	0·874 0·874
2	Two right-angle bends in opposite directions	12·8 20	3·21 3·99	24·1 24·5	24·3 24·3	7·31 7·31	0·79 0·79	1·748 1·748
3	One bend at 45 deg.	12·8 20·8	3·56 4·42	19·7 20·8	20·25 20·25	3·68 3·68	0·398 0·398	0·874 0·874
4	Two bends a 45 deg. in opposite directions	12·8 20·8	3·55 4·16	19·95 20·3	20·1 20·1	3·39 3·39	0·368 0·368	1·748 1·748
5	One right-angle bend	15·2 24·8	1·76 2·17	96·5 104	100 100	12·14 12·14	0·31 0·31	0·398 0·398
5*a*	One right-angle bend	15·2 24·8	1·73 2·20	100 101	100·5 100·5	11·14 11·14	0·26 0·26	0·148 0·148
6	Two right-angle bends in opposite directions	15·2 24	1·66 2·10	109 108	108·5 108·5	17·5 17·5	0·45 0·45	0·546 0·546
6*a*	Two right-angle bends in opposite directions	15·2 24	1·70 2·11	104 107	105·5 105·5	14·5 14·5	0·37 0·37	0·546 0·546
7	One bend at 45 deg.	14·4 22·4	1·74 2·16	94 94	94 94	7·70 7·70	0·20 0·20	0·398 0·398
7*a*	One bend at 45 deg.	14·4 22·4	1·81 2·23	87 88·5	87·5 87·5	0·50 0·50	0·013 0·013	0·148 0·148
8	Two bends at 45 deg. in opposite directions	15·2 24	1·85 2·26	88 93	90·5 90·5	1·80 1·80	0·04 0·04	0·546 0·546
8*a*	Two bends at 45 deg. in opposite directions	15·2 24	1·76 2·24	97 95	96 96	7·30 7·30	0·18 0·18	0·546 0·546

of curvature ρ. The principal difficulty in making such experiments as these seems to be that the resistance of the bend is only a small part of the whole, and therefore, as it is taken as the remainder when the rest are deducted, whatever errors may creep in are included in it.

CHAPTER XI.

Experiments by M. Lelong on Various Types of Fans— The first fan tested was one designed for the Du Chayla. Its dimensions were the following: External diameter of wheel, 5·25 ft.; diameter of eye, 3·28 ft.; number of vanes, 24; width of vanes at the outer circumference, 0·492 ft. The casing was a volute whose sections were calculated by the formula

$$s = \frac{Q \cdot \theta}{v}$$

where Q = cubic feet per second, θ = angle measured from the commencement of the volute to the section, v the absolute velocity of outflow from the fan, and s the section

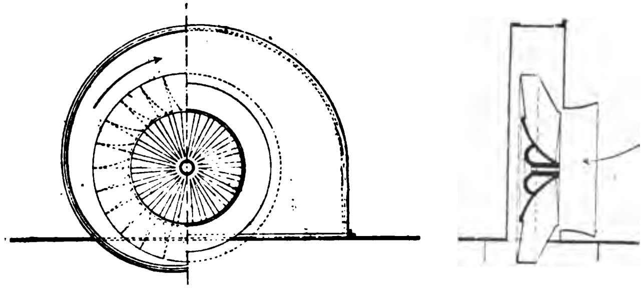


Fig. 56*a*.

of the volute in square feet. The fan is shown in fig. 56*a*. This fan was first tested without the inflow mouthpiece, and curve (1), fig. 56*b*, shows its characteristic, the ordinates being manometric efficiencies, and the abscissæ reduced orifices

$$\frac{Q}{r_1^2 \sqrt{2\,g\,H}},$$

the 2 in the denominator having been lately added by M. Rateau. With the mouthpiece the characteristic was (2). Fig. 56*c* is the second fan tested, having an external diameter of wheel, 5·57 ft.; diameter of eye, 3·64 ft.; number of

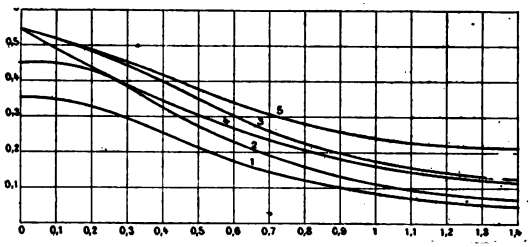


FIG. 56*b*.

vanes, 16; width of vanes at the external diameter of the wheel, 0·574 ft. The volute is now calculated by the formula

$$s = \frac{1 \cdot 8 \, Q \, . \, \theta}{v}.$$

This volute receives the air discharged by about three-quarters of the disc. It ends in a pyramidal chimney

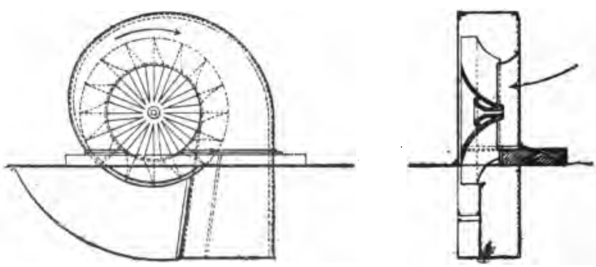


FIG. 56*c*.

having an angle of inclination of faces of 7 deg., whilst the air discharged from the last quarter of the disc is received by a diffuser. The air leaves the chimney and diffuser with a velocity equal to half that from the wheel. The characteristic of this fan is (3), fig. 56*b*. Curve (4) shows the effect of doing away with the diffuser. These two fans had vanes ending radially, and M. Lelong considers that their manometric efficiency is the same as the mechanical efficiency of the fan. This, however, is doubtful. The latter is probably more, because it is quite possible that $w_1 < c_1$ as the angle of flow is not always the same as the angle of vane.

The third fan tested, fig. 56*d*, had two eyes. Its vanes were inclined forward at 45 deg. to the radius. The sections of the volute were the same as in the last case, but there was no diffuser, and this we believe is the reason that the manometric efficiency was only about 65 per cent at most. The external diameter of the vanes was 4·59 ft., the diameter of each eye 0·295 ft.; the number of vanes was 16, and their width at the outer diameter of the wheel ·459 ft. The characteristic curve of this fan (6), fig. 56*e*, is much higher than the last, but its mechanical efficiency is not any greater than that of the first or second. If we assume that the angle of relative outflow from the wheel is the angle of the vane, the efficiency of the fan alone in this case should be very nearly

$$\eta = \frac{g\,H}{c_1\,w_1} = \frac{\sqrt{M}\,\sqrt{g\,H}}{c_1 + u_1} = \frac{\sqrt{M}}{\frac{1}{\sqrt{M}} + \frac{u_1}{\sqrt{g}\,H}}$$

$$= \frac{M}{1 + \frac{Q\sqrt{M}}{\sqrt{2\,g\,H}\,\sqrt{2}\,\pi\,r_1\,b_1}} = \frac{M}{1 + \frac{\phi\sqrt{M}\,r_1}{b_1\,\pi\sqrt{2}}}$$

$$= \frac{M}{1 + 1{\cdot}126\,\phi\sqrt{M}}$$

where
$$\phi = \frac{Q}{r_1^{2}\sqrt{2\,g\,H}}$$

which is called the reduced orifice, the $\frac{1}{\sqrt{2}}$ having been lately added by Professor Rateau.

This gives us from curve 6 the following table :—

Reduced orifice ϕ ...	·2	·3	·4
η	·534	·497	·451

which shows very clearly the mistake of not having a proper diffuser into which the wheel might discharge before the air

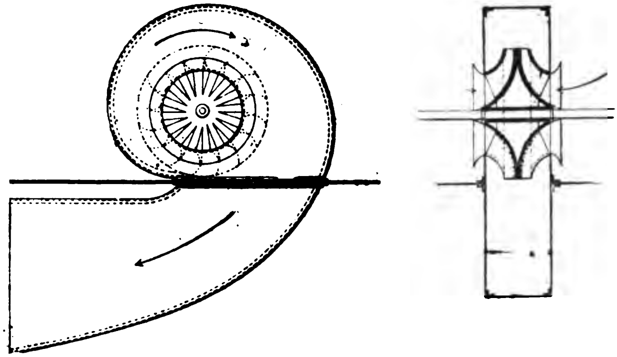
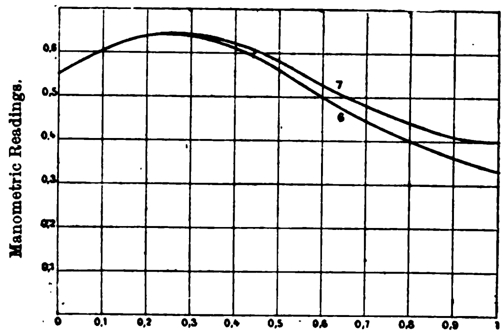


FIG. 56d.

entered the volute. Curve 7 shows the dynamic mano-metric efficiency of the fan. Fig. 56f shows an open running fan with radial vanes tested with a casing. Its



Orifices.

FIG. 56e.

dimensions were: Diameter of wheel, 4·42 ft.; diameter of eye, 2·62 ft.; number of vanes, 34; breadth of wheel at discharge, ·354 ft. It gave the characteristic curve 8. It was tested without any casing. Curves 9 and 10 (fig. 56*g*) represent the characteristics with and without inflow mouthpiece of a fan constructed for experimental purposes, and only differing from the preceding in the inclination of

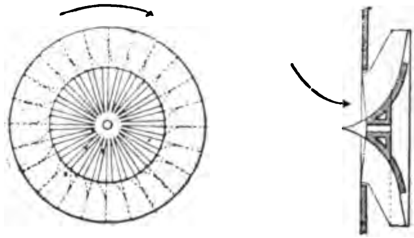


FIG. 56*f*.

its vanes at the outer radius (fig. 56*h*). Its manometric efficiency was less than that of the preceding, as its vanes curved backwards. If we increase the height of curve 8 in the same proportion as 9 is above 10, we get curve 11,

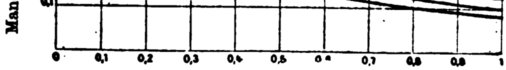


FIG. 56*g*.

which M. Lelong considers would have been the characteristic of the fan (fig. 56*f*) if an inflow mouthpiece had been added.

The Mortier Diametral Fan.—In fig. 57 is shown a Mortier diametral fan. The direction of rotation is counter

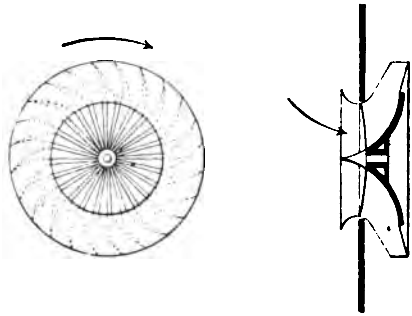


FIG. 56h.

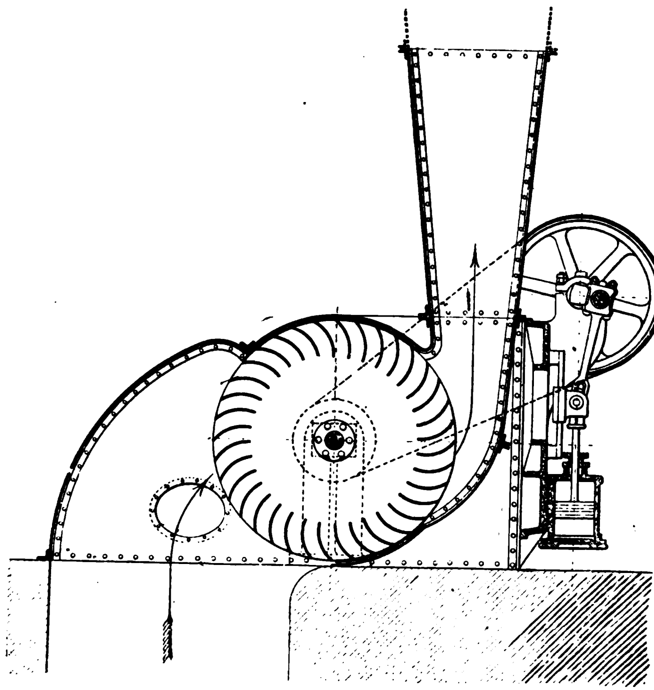


FIG. 57.

clockwise, and the air enters and leaves as shown by the arrows. The forward inclination of the vanes allows of inflow without shock, and the air at outflow is thrown forwards as well as outwards. Its velocity head, which must then be considerable, is converted by the chimney into pressure head. These fans are made by Mr. Louis Galland, of Chalon-sur-Saône. The results of experiments with a fan of this type, of 6·56 ft. diameter by 3·94 ft. broad, are shown

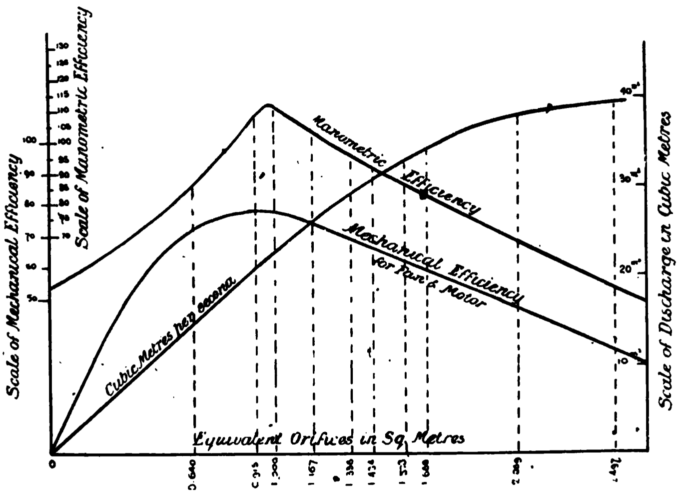


FIG. 58.

in fig. 58, in which are given the mechanical efficiency of fan and engine, the manometric efficiencies, and the quantity that would be discharged per second in cubic metres at 225 revolutions per minute, or a peripheral velocity of 77·4 ft. per second.

In order to get from this curve the volumetric efficiency $Q \div c_1 r_1^2$ the reader may divide by 23·6. We have, however, already explained that we do not consider this formula a good comparison between weight, size, and tip speed of fan. The volumetric efficiency at the orifice at which the maximum mechanical efficiency was obtained is very nearly 89 per cent, and is very high.

We are not prepared to theorise as to what happens within the eye of the wheel. The vanes are radial at the inner circumference of the wheel, and it may be that in passing across the eye the air moves in such a curve that its

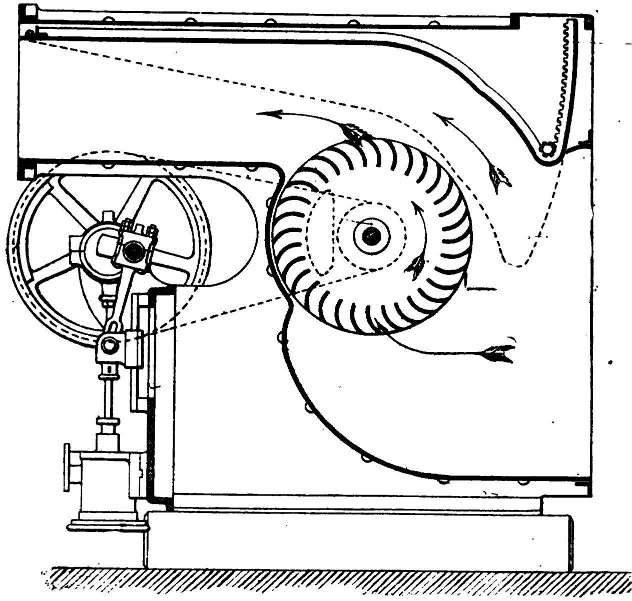


FIG. 59.

radial velocity is unaltered, and its tangential is reversed, so that it enters the wheel again without shock; but this seems very doubtful. Also the fact that the motion in each passage is begun, stopped, and reversed, and stopped again, every revolution, must be the cause of waste of energy, but of how much it is difficult to say.

These fans can by an adjustment of the casing be arranged so as to work at variable orifices. For example, if the orifice of a mine increases, the casing of the fan can be moved away from the wheel, fig. 59, by the rack and pinion from the position shown by the dotted lines, and it will, according to the makers, discharge more air with equal efficiency.

Fig. 60 shows a larger fan, in which the position of the casing is permanently moved outwards from its original position, as shown by the dotted lines.

The Kley Ventilator.—In this fan (figs. 61 and 62) the vanes are radial and plane, but two spiral inflow passages are provided in which the air obtains tangential motion in the same direction as that in which the wheel is running, which is clockways, in fig. 62. The suction passage is so calculated that in normal working the air will enter the wheel without shock. After leaving the wheel it enters the volute, whose section increases gradually towards the discharge. It will be noticed here that the work done by the wheel per pound of air is

$$\frac{1}{g}(c_1^2 - c_2^2)$$

or

$$\frac{1}{g}c_1^2\left(1 - \left(\frac{r_2}{r_1}\right)^2\right)$$

They are constructed with inflow at one or both sides; for forges and foundries from 11·8 in. to 7·2 ft. diameter, and for mine ventilation from 16·4 to 39·3 ft. diameter. The following table gives a series of experiments upon a

Revolutions of fan per minute.	Water gauge in the suction pipe in inches. h	Cubic feet of air per second. Q	Approximate manometric efficiency per cent assuming $H = \frac{10000}{144}h$. μ	Mechanical efficiency of engine and fan per cent.
30	·86	493	89·60	..
40	1·35	622	79·25	54
50	1·97	751	74·00	..
60	2·85	881	74·50	56
70	3·84	1,020	73·50	..
72	4·10	1,055	74·2	58

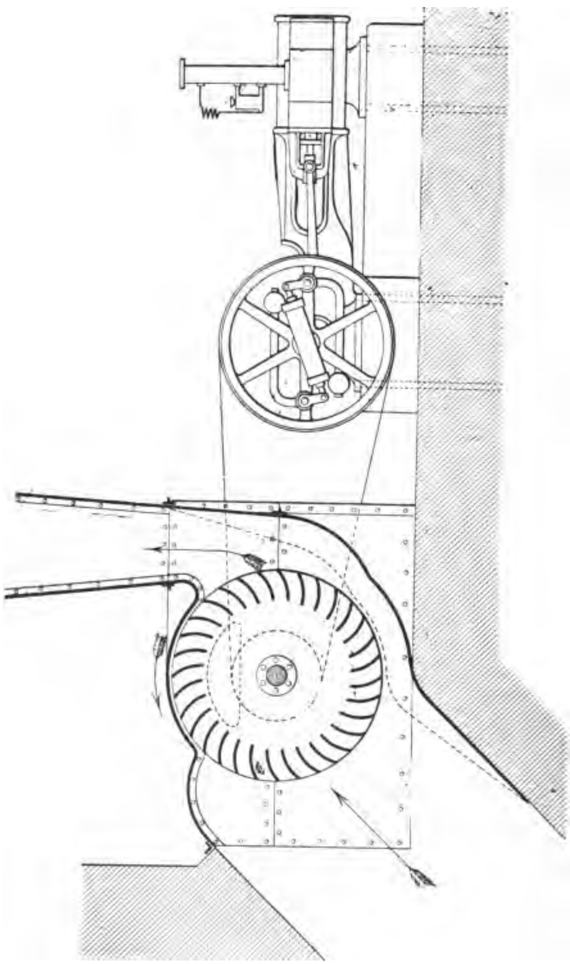


FIG. 60.

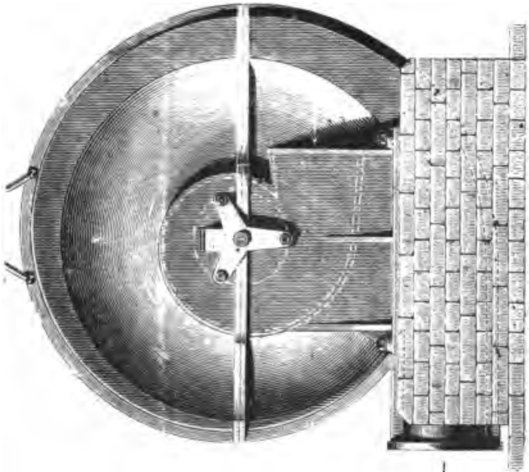


FIG. 62.

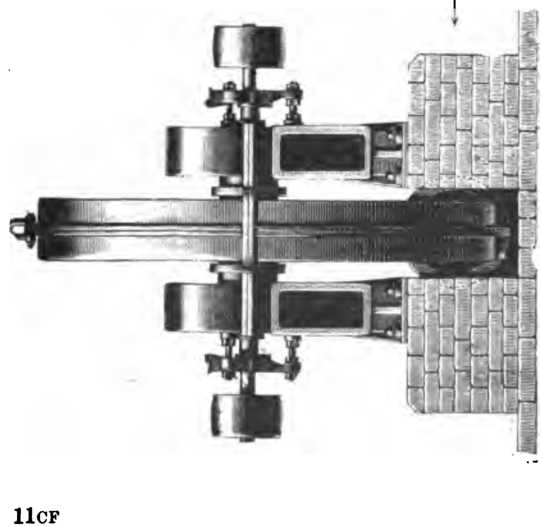


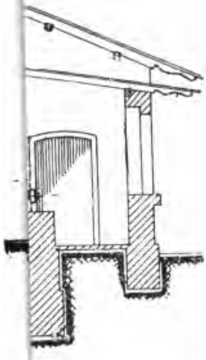
FIG. 61.

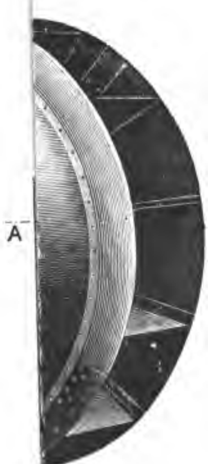
ventilator of this type of the following dimensions: External diameter, 29½ ft.; internal diameter, 19·65 ft.; external breadth, 2·62 ft.; internal breadth, 3·94 ft.

The following table, for which, with figs. 61 and 62, we have to thank Mr. C. Mehler, of Aachen, give dimensions, &c., of the smaller fans of this type.

No. of fan.		Water gauges in inches.					Diameter of wheel in inches.	Diameter of discharge pipe in inches.
		7·89	11·8	15·75	19·70	23·60		
1	Discharge in cubic feet per minute.	705	880	1,055	1,230	1,410	29·6	8·87
	Corresponding revolutions.	1,289	1,569	1,810	2,026	2,216		
2	Discharge in cubic feet per minute.	1,230	1,410	1,760	1,930	2,110	39·4	11·8
	Corresponding revolutions.	960	1,177	1,358	1,520	1,662		
3	Discharge in cubic feet per minute.	1,760	2,110	2,640	3,000	3,170	49·25	14·8
	Corresponding revolutions.	768	942	1,086	1,216	1,330		
4	Discharge in cubic feet per minute.	2,810	3,520	4,230	4,760	5,290	61·5	18·4
	Corresponding revolutions.	615	755	870	974	1,065		
5	Discharge in cubic feet per minute.	5,640	6,700	8,450	9,510	10,550	87·5	26·2
	Corresponding revolutions.	..	..	..	..	..		

The Pelzer Fan.—This is shown in figs. 63, 64, 65, and 65A. It is a type of fan largely used on the Continent. It has in late years been greatly improved. Fig. 65A shows the wheel. The eye is to the left, and in it are seen twelve vanes of a curved form, which receive the air without shock as it enters flowing parallel to the axis. Having passed these, it is received by vanes which are plane and radial, and a manometric efficiency of 50 per cent is generally obtained,




A

but this by alterations in the construction can be considerably increased.

Figs. 63, 64, and 65 show the diffuser and volute, an iron chimney, and the plant for driving the fan. The aerial pressure on the wheel is balanced by allowing the air from the diffuser to flow into the conical spaces surrounding the left side of the circumference of the wheel. A thrust, fig. 65, to the right is obtained to balance the thrust that naturally acts towards the left or suction side.

These fans are made with diameters between 11·8 in. and 19·7 in., while the breadths of the wheel are—

$\frac{8}{10}$ths	of the	diameter	for a water gauge of	0·4	inch.
$\frac{8}{10}$ths	”	”	”	0·8	”
$\frac{7 \cdot 5}{10}$ths	”	”	”	1·2	”
$\frac{7 \cdot 5}{10}$ths	”	”	”	2	”
$\frac{7}{10}$ths	”	”	”	3	”
$\frac{7}{10}$ths	”	”	”	4	”
$\frac{7}{10}$ths	”	”	”	6	”
$\frac{7}{10}$ths	”	”	”	8	”
$\frac{6}{10}$ths	”	”	”	10	”
$\frac{6}{10}$ths	”	”	”	12	”
$\frac{6}{10}$ths	”	”	”	14	”

These fans are made by Mr. F. Pelzer, Dortmund.

The Bumstead and Chandler Fan.—This is shown in figs. 66 and 67 in sectional plan and elevation. There are two engines, one on each side of the fan, which can be run separately or both coupled to the fan. They have high-pressure cylinders, 16 in. diameter, and low-pressure, 24 in.; they are tandem engines, with a stroke of 16 in. These engines when running at 220 revolutions indicate 320 horse power. The fan is 15 ft. in diameter and 6 ft. 6 in. wide, and is capable of discharging 250,000 cubic feet of air per minute, but as it was found impossible to get this quantity through the mine at the stipulated water gauge in the fan drift, it was decided to permit some air to enter the top of the upcast shaft, and then to measure the total volume in the fan drift, which is of ample area and length to obtain accurate measurements. The air from the mine enters the fan at both sides, fig. 67 (which is a sectional elevation through the centre of the fan), and is discharged upwards to the atmosphere after leaving the fan. The chimney is very large, in order that the air may be discharged at low velocity. The casing is a volute. The blades of the fan are mounted on a steel disc, 10½ ft. diameter.

The makers have made many experiments to find the best form of blade, and have proved to their own satisfaction that a modified S form is the best, with the inner end of the

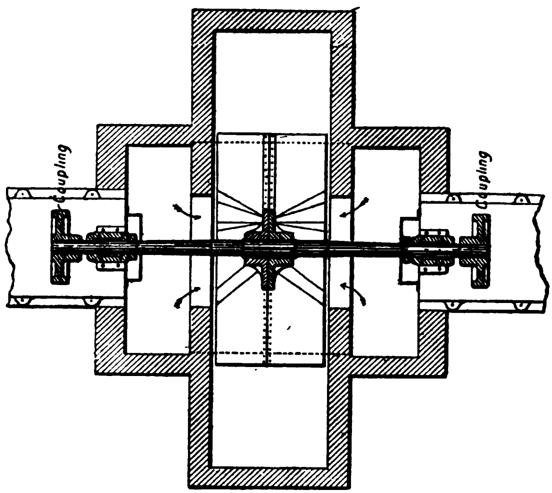


FIG. 66

blade curved forward in the direction of rotation, so as to cut into the air, and gradually raise its velocity as it passes outwards.

It will be noticed that the fan-shaft bearings are not in the air drift in the usual way, but are isolated therefrom by a sheet-steel cover. The bearings are thus practically in the engine room, and free from all dirt and dust that passes through the fan. These bearings are adjustable vertically by means of taper wedges, adjustable by screws.

The following is a series of tests upon the above fan by Mr. Strick, manager of the Cossall Colliery. The equivalent

EXPERIMENTS WITH A BUMSTEAD AND CHANDLER FAN AT COSSAL COLLIERY.

No. of test.	Revolutions per minute.	Water gauge in the fan drift 30 ft. from the fan.	Volume passed through the fan drift, cubic feet per second.	Useful work done by fan in H.P.	I.H.P. of the engines.	Efficiency of engine and fan.	Fan blade tip speed in feet per second.	Manometric efficiency per cent.	Equivalent orifice.
1	150	2·8	2860	75·9	106·5	71·28	117·8	45·0	39·65
2	151	3·1	2885	84·5	117·2	72·1	118·6	49·1	37·9
3	204	5·2	3470	170·6	260·2	65·5	160·2	45·3	35·2
4	205	4·75	4100	184·3	261·4	70·5	161·0	41·0	43·6
5	204	5·5	3580	186·4	263·5	70·74	160·2	48·0	35·4
6	200	4·5	3885	165·1	230·0	71·5	157·0	40·7	42·4
7	202	2·8	4650	....	....	....	158·6	24·8	64·2
8	203	3·1	4650	....	....	....	159·4	27·4	61·0

orifices and manometric efficiencies have been added. The value of H has been calculated by the formula—

$$H = \frac{10000}{144} h.$$

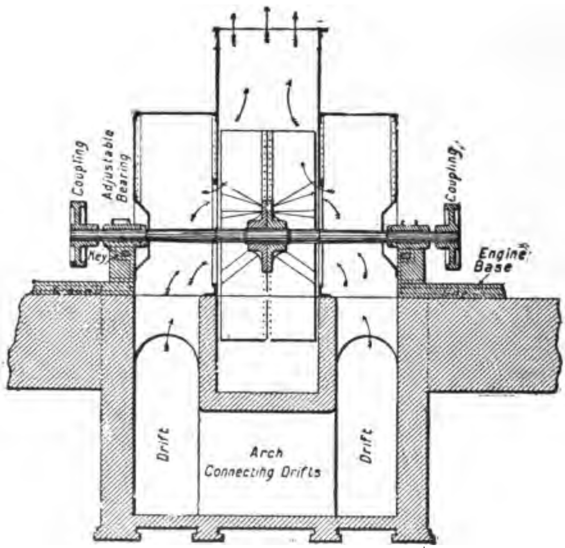


FIG. 67.

The Ser Fan.—This has already been illustrated and described in figs. 50 and 51. We add here some experiments with a small blowing fan of the following dimensions:—*

External diameter .... 19·7 in.
Internal diameter .... 11·8 in.
Length of vanes radially .... 3·94 in.
Width of vanes parallel to the axis ...... 3·55 in.
Cross-section of the discharge pipe...10·25 × 9·84 in.
= 0·7 sq. ft.

* From "Die Gebläse," by von Ihering.

Revolutions of fan per minute.	Water gauge in inches. h	Manometric efficiency per cent.	Horse power measured at the dynamometer.	Useful H.P. of fan.	Mechanical efficiency of fan alone.	Cubic feet of air per second. Q	Equivalent orifice in square feet. $\frac{Q}{43 \cdot 4 \sqrt{}}$
1292	5·26	93·5	7·8	4·96	63·6	99·5	1·0
1094	3·69	91·0	4·65	2·91	62·6	83·2	·975
1002	3·16	93·0	3·45	2·31	67·0	77·0	·996
830	2·22	95	2·37	1·35	57	64·5	·995

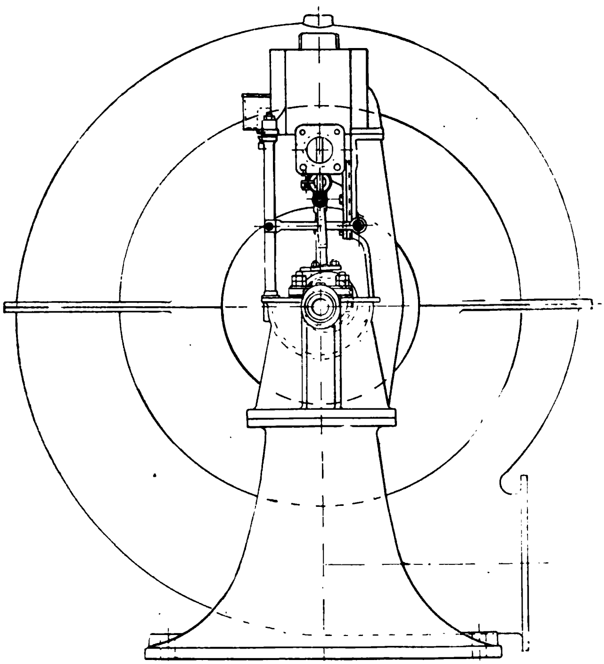


FIG. 68.

A transmission dynamometer must have been used here, as the mechanical efficiency of the fan alone is given. These experiments are an excellent illustration that the manometric and mechanical efficiencies are very nearly constant at a constant equivalent orifice. The friction of the shaft probably accounts for the fall in the latter efficiency, except in the third experiment.

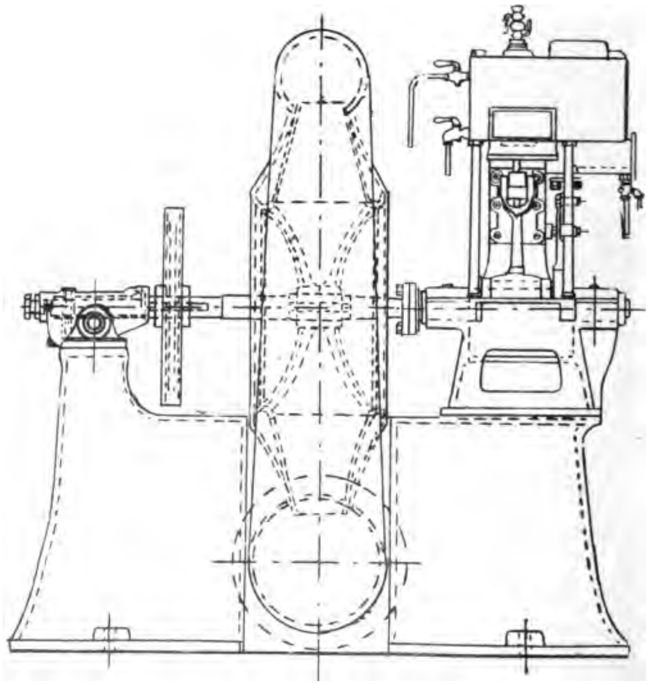


FIG. 68A.

Beck and Henkel Fans.—Figs. 68 and 68A show a type of fan made by Messrs. Beck and Henkel, Cassel. It is driven direct from a small vertical engine. It has a cast-iron spiral casing, and inflow takes place at both sides; the casing is formed in two halves, so that the wheel is readily accessible. The centre of the wheel is formed of cast iron, having a number of arms connecting the boss to two rings. To these, by means of angle irons, vanes of sheet steel are fastened, and the shaft is carried on the left of fig. 68A in a self-adjusting bearing, having a bearing surface

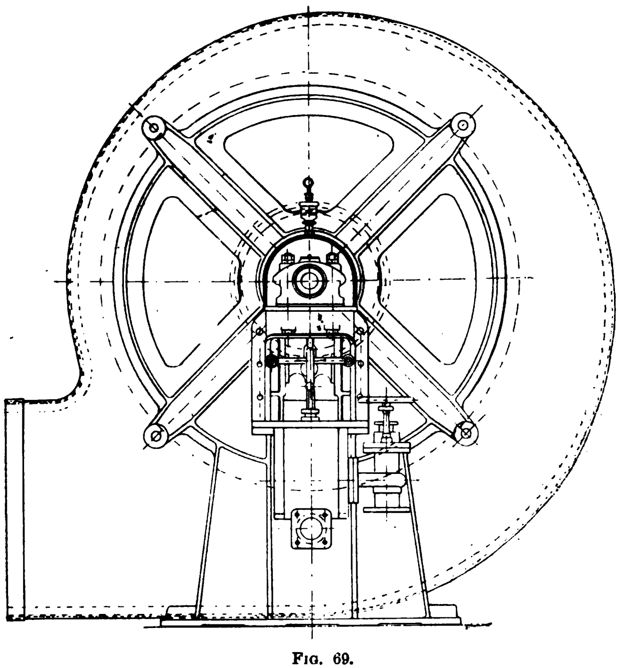


FIG. 69.

of white metal and an oil chamber at the left beneath it, from which oil can be raised by a ring carried by the shaft. These fans can be driven direct by engine or electro-motor, or by belt and pulley. These fans are made with wheel diameter between 11·8 in. and 59·1 in., and can supply air at 29·5 in. of water gauge. They can therefore be employed for forges, cupolas, &c.

Figs. 69 and 70 show another fan made by the same firm, with inlet at one side and a sheet-iron casing; these are for

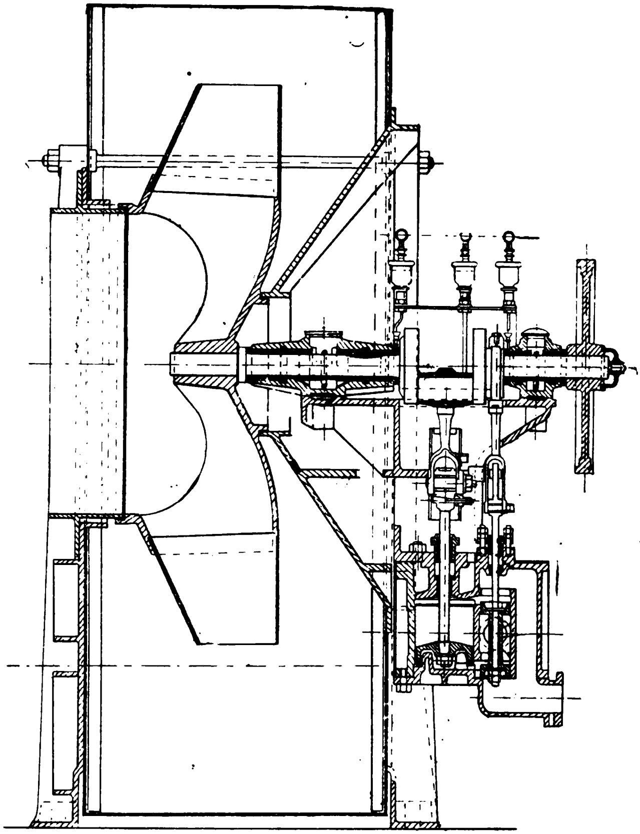
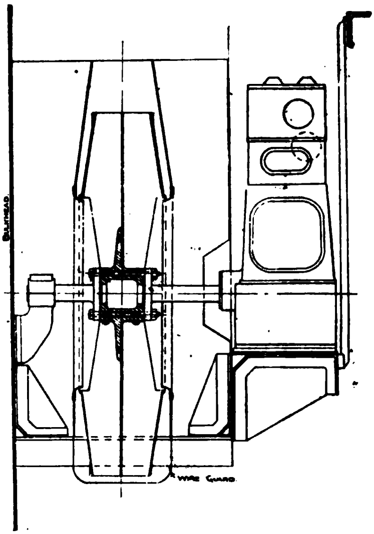


Fig. 70.

producing moderate pressures, and for the discharge of large quantities of air. The centre of the wheel is of cast iron, but to this are fixed sheet-iron vanes. This type can be driven direct or by pulley, and are made with wheel diameters between 11·8 in. and 118 in.

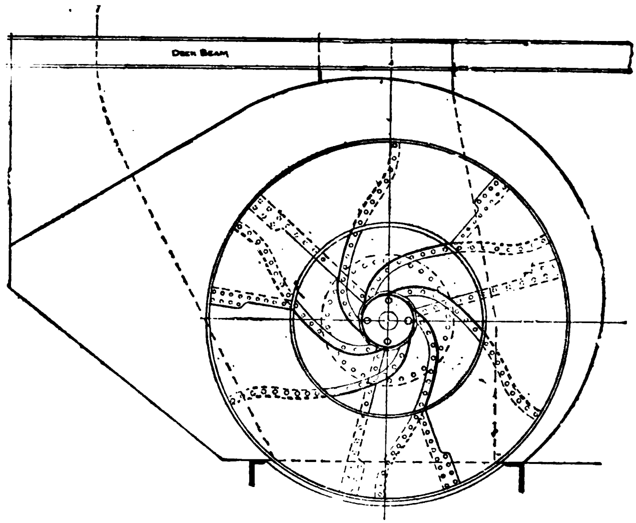
Figs. 71 and 72 give two sectional views of a double inlet fan constructed by Messrs. W. H. Allen, Son, and Co., of Bedford. It is here represented as fixed on



66 in. Double Inlet Forced Draught Fan.

FIG. 71.

board ship. Its diameter is 66 in., and it is intended to produce a forced draught, and its normal output is 25,000 cubic feet per minute, but this can be varied according to the work it has to do. Its vanes are connected to the conical casing by means of angle irons. They at first curve backwards in the opposite direction to that in which the wheel rotates, but as they near the outer periphery they have a contrary curvature, ending radially. The wheel is driven direct by one of Messrs. Allen's enclosed forced lubrication engines. The introduction of the enclosed type of engine for this purpose is to enable the plant to run continuously at high speed without much attention. These fans are also supplied

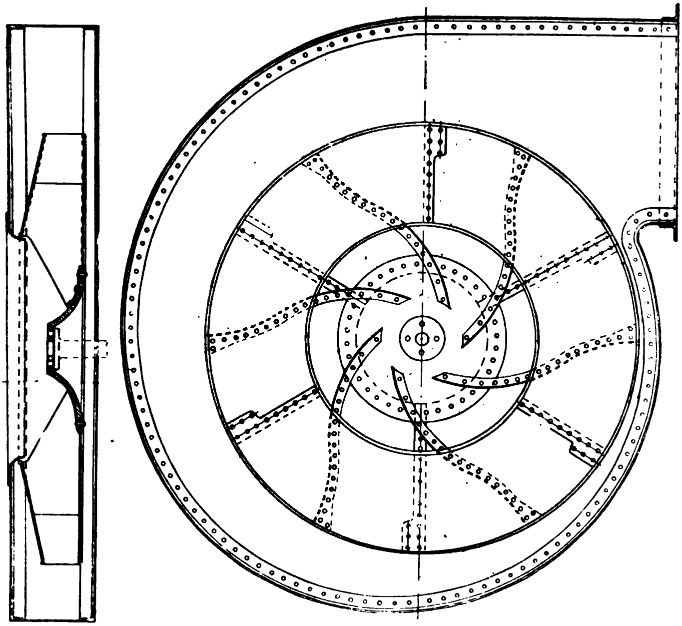


66 in. Double Inlet Forced Draught Fan.

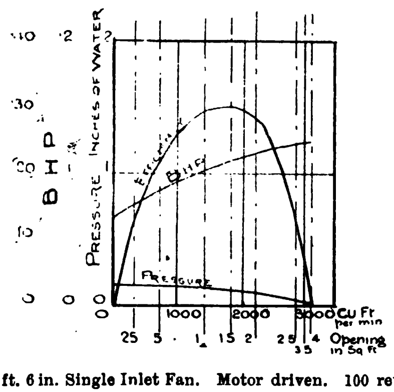
FIG. 72.

with electro-motors, but in the past the majority of them have been steam driven. It will be noticed that only part of the circumference of the wheel is surrounded by a volute, the remainder discharging into a diffuser.

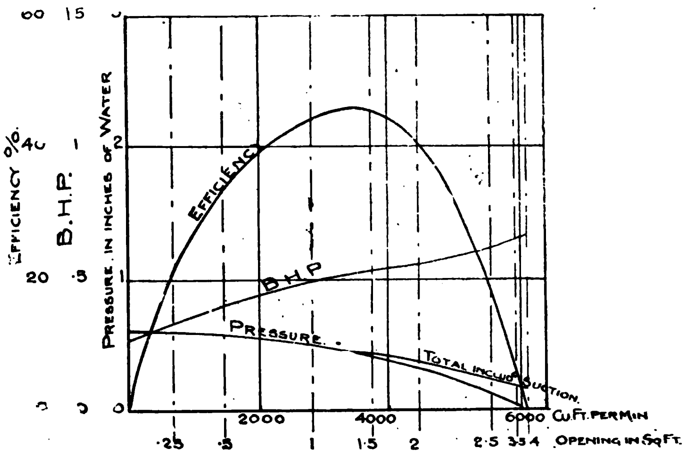
Figs. 73 and 74 give a sectional front and side elevation of a 78 in. single inlet fan by the same makers. The wheel centre is of cast iron, but the rest of the wheel casing and vanes are of wrought iron. These latter are of the same form as is the previous fan, but the wheel is completely



FIGS. 73 AND 74.—78 in. Single Inlet Fan.

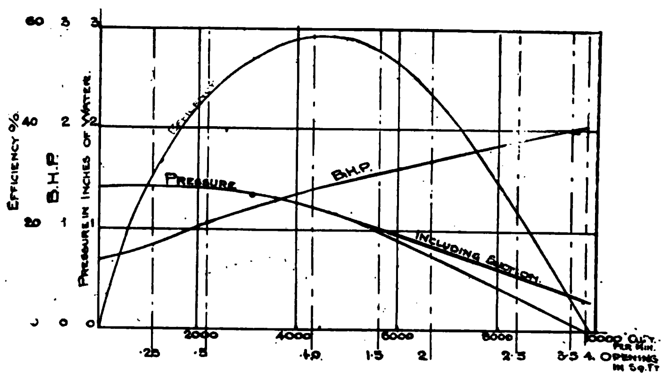


Tests of 4 ft. 6 in. Single Inlet Fan. Motor driven. 100 revolutions.
FIG. 75.



Tests of 4 ft. 6 in. Single Inlet Fan. Motor driven. 200 revolutions.

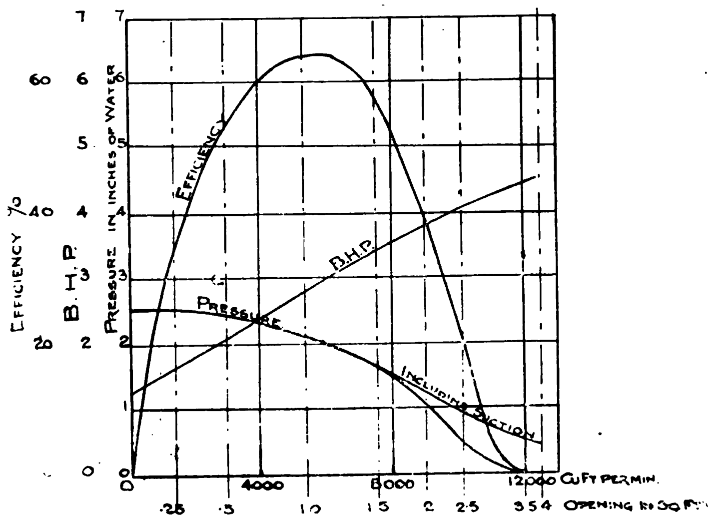
Fig. 76.



Tests of 4 ft. 6 in. Single Inlet Fan. Motor driven. 300 revolutions.

Fig. 77.

surrounded by a volute. The normal capacity of this fan is about 17,500 cubic feet per minute, and it can be driven either electrically or by steam. Figs. 75 to 82 are curves made from tests with a 4 ft. 6 in. diameter single inlet fan similar to the last. They give the water gauge, brake horse power, and efficiency as ordinates, with the number of cubic feet per minute as abscissæ. We give these curves taken at



Tests of 4 ft. 6 in. Single Inlet Fan. Motor driven. 400 revolutions.

Fig. 78.

from 100 to 800 revolutions per minute, because they are almost a conclusive proof of the theory previously given in these pages. In the first place, it will be noticed that the cubic feet per minute when the water gauge is zero is a little more than thirty times the number of revolutions per minute, and if we calculate the equivalent orifices when the discharge in cubic feet is fifteen times the revolutions per minute, we obtain the following table :—

Revolutions per minute	200	300	400	600	700	800
Cubic feet per minute	3,000	4,500	6,000	9,000	10,500	12,000
Water gauge in inches	0·5	1·2	2	4·6	6·1	8·3
Equivalent orifice	1·63	1·58	1·605	1·614	1·63	1·566

The equivalent orifice is $\frac{Q}{\cdot 65 \sqrt{2\,g\,H}}$ where Q is cubic feet per second and H is head of air in feet. Putting $Q = \frac{Q_1}{60}$ where Q_1 = cubic feet feet per minute, and

$$H = h \times \frac{10000}{144}$$

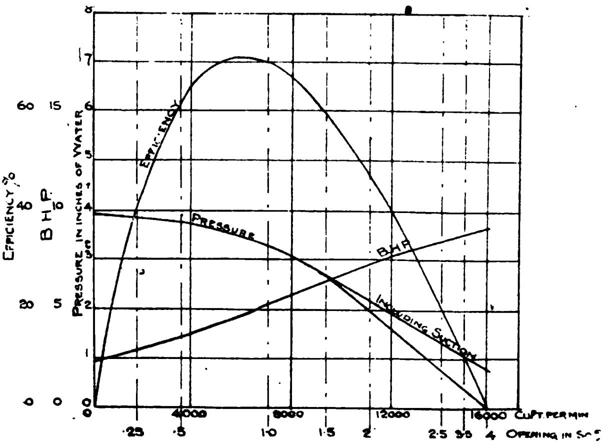
which gives a correct value for the average densities of air and water, we obtain the equivalent orifice

$$O = \frac{Q_1}{2600 \sqrt{h}},$$

from which the values in the above table are calculated. This shows that when $Q \propto c_1$ tip speed $Q \propto \sqrt{h}$; it follows that the manometric efficiency at this equivalent orifice is constant. It can be calculated by the formula

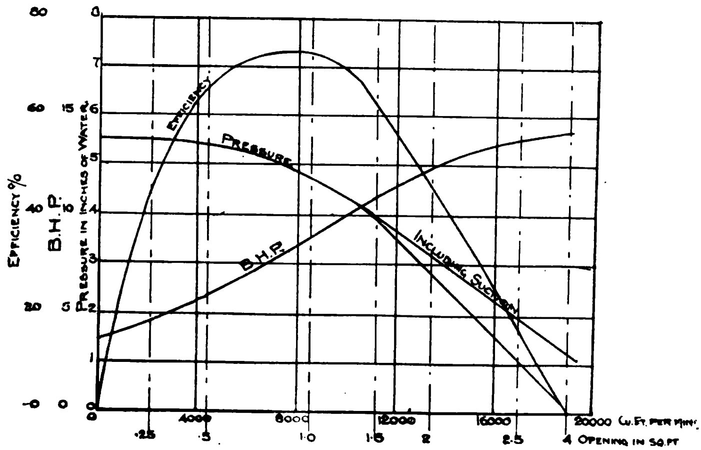
$$M = \frac{g\,H}{c_1^2} = \frac{40100\,h}{(\text{revolutions per minute})^2}$$

and we obtain values between 52 and 50·2 per cent at this orifice. The mechanical efficiency decreases with the speed as the brake horse power of the motor that drives the fan is partly absorbed by the work done on the air by the wheel, and by the surface friction of the outside of the wheel, two quantities of work which vary as the cube of the revolutions, but in addition to this there is the bearing friction, and this appears to require work roughly proportional to the revolutions. For if R is the number of hundreds of revolutions per minute, and we assume



Tests of 4 ft. 6 in. Single Inlet Fan. Motor driven. 500 revolutions.

Fig. 79.



Tests of 4 ft. 6 in. Single Inlet Fan. Motor driven. 600 revolutions.

Fig. 80

$$\text{B.H.P.} = A\,R^2 + C\,R,$$

or

$$\frac{\text{B.H.P.}}{R} = A\,R^2 + C,$$

then for 800 and 400 revolutions we get

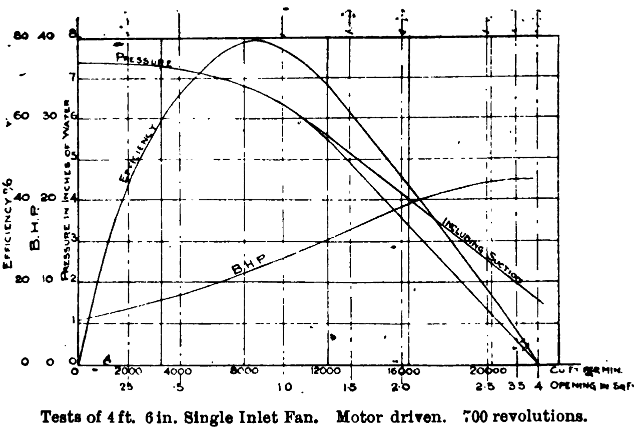
$$\frac{20}{8} = 64\,A + C$$

$$\frac{6}{8} = 16\,A + C,$$

$$\text{so that } A = \frac{7}{192} \text{ and } C = \tfrac{1}{6}.$$

This gives us the following table :—

B.H.P. actual	13·70 ...	8·50 ...	1·50 ...	·500
B.H.P. calculated	13·66 ...	8·875 ...	1·48 ...	·625
Revolutions per minute.	700 ...	600 ...	300 ...	200



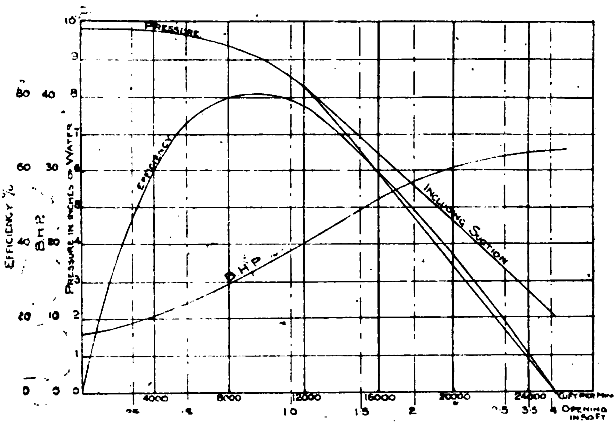
Tests of 4 ft. 6 in. Single Inlet Fan. Motor driven. 700 revolutions.

FIG. 81.

which shows a very close agreement, and implies that the journal friction force is constant and the work required to overcome it varies as the revolutions. The useful work varies as the cube of the revolutions for Q $\propto \sqrt{h} \propto$ revolutions at a given orifice and the useful work $\propto$ Q h, and therefore $\propto$ (revolutions)^3. The air efficiency

$$\eta = \frac{\text{useful work done}}{\text{work transmitted to wheel}}$$

is therefore probably a constant quantity.



Tests of 4 ft. 6 in. Single Inlet Fan. Motor driven. 800 revolutions.

FIG. 82.

The Geneste-Herscher Fan.—This is of considerable interest, as it gained the first prize at the Paris Exhibition of 1900. A small fan of 2·62 ft. diameter is shown in figs. 83 and 84, and a diagram, fig. 85, shows curves of mechanical efficiency of motor and fan, manometric efficiency, and a curve of volumes in cubic metres, which must be multiplied by 35·32 to reduce to cubic feet, and the depression in millimetres must be divided by 25·4 to reduce to inches. The tests were made at the Blanzy Mines. The fan had a diameter of 4·75 ft., and was driven by two engines having diameters of cylinders of 18·65 in., and a stroke of 27·5 in. The curves are reduced to what would have been obtained at 545 revolutions, corresponding to a tip speed of 135·5 ft. per second. The

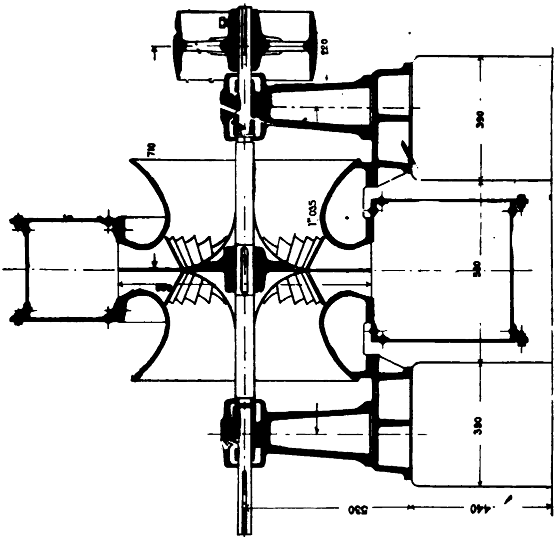


FIG. 84.

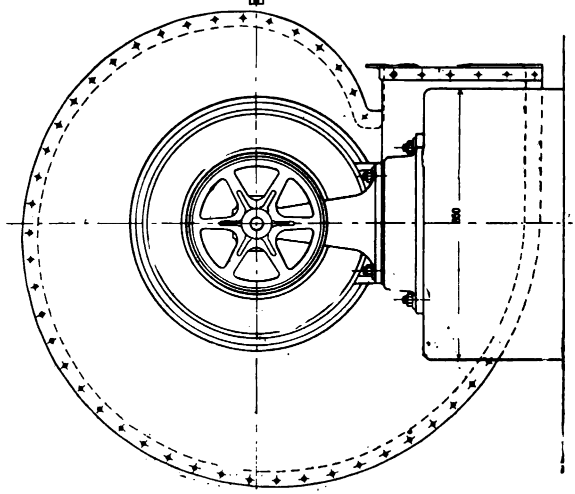
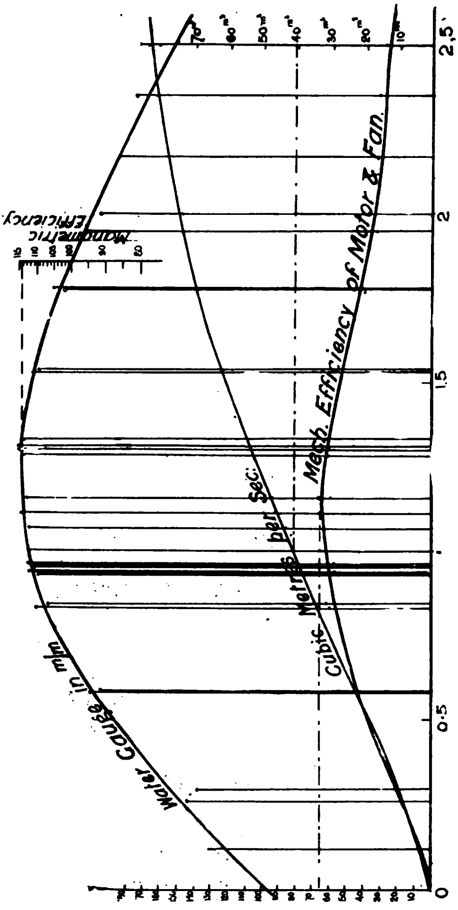


FIG. 83.

Scale of Discharge in Cubic Metres.



Scale for Water Gauge and Mechanical Efficiency.

FIG. 85.
Equivalent Orifices in Square Metres.

highest mechanical efficiency was 66 per cent, and was obtained at an equivalent orifice of about 12·35 square feet; while the highest manometric efficiency, 114·5 per cent, was obtained at an orifice of 13·45 square feet—two values very close together. The curve of volumes will give volumetric efficiency, if divided by $c_1 r_1^2$ in metric units, or 21·7. The fan of 2·62 ft. diameter—figs. 71 and 72—has cylindrical vanes, and a cylindrical section of the wheel first decreases from the eye radially, and then again increases from about the middle of the blade to the circumference. There is no diffuser, but the air is discharged into a volute of rectangular section. The vanes must be curved forwards considerably to obtain the high manometric efficiency. The breadth of wheel at outflow is 7 in., and at the inner radius of the vanes 8·3 in. The external diameter is 31½ in., and the internal at the corners of the vanes 20 in. The mouthpiece at the suction has a diameter of 28 in. The volute appears to have a section increasing uniformly with the angle from the beak, and at discharge is 17·6 in. by 17·1 in. Fans for mine ventilation have, of course, a chimney. Assuming a mechanical efficiency of 70 per cent for this type of fan, with a manometric of 110 per cent, and an equivalent orifice,

$$O = \frac{9}{10} \times 12{\cdot}35 \times \frac{(2{\cdot}62)^2}{(4{\cdot}75)^2} = 3{\cdot}25 = \frac{Q}{{\cdot}65\sqrt{2gH}}$$

which gives the same reduced orifice as that of the Blanzy fan, multiplied by $\frac{9}{10}$ to allow for exaggeration by the anemometer. Then

$$\frac{gH}{c_1 w_1} = {\cdot}70$$

$$\sqrt{M}\,\frac{\sqrt{gH}}{c_1 - u_1 \cot\phi} = {\cdot}70$$

$$\frac{1{\cdot}05}{{\cdot}70} - \frac{c_1}{\sqrt{gH}} = \frac{u_1}{\sqrt{gH}} \cot\phi$$

$$\frac{1{\cdot}50 - {\cdot}95}{\dfrac{u_1}{\sqrt{gH}}} = \cot\phi.$$

But

$$u_1 = \frac{Q}{2\pi r_1 b_1} = \frac{0 \times \cdot 92 \sqrt{g\,H}}{\pi \times 2\cdot 62 \times \cdot 584}$$

$$= \cdot 622 \sqrt{g\,H}$$

$$\therefore \cot\phi = -\frac{\cdot 45}{\cdot 622} = -\cdot 722$$

$$\phi = 125^\circ - 50'$$

the angle of outflow.

The velocity of whirl $w_1 = c_1 - u_1 \cot\phi$

$$= (\cdot 95 + \cdot 449)\sqrt{g\,H} = 1\cdot 399 \sqrt{g\,H}.$$

That in the volute

$$v_4 = \frac{144\,Q}{17\cdot 6 \times 17\cdot 1}$$

$$= \frac{144 \times 3\cdot 25 \times \cdot 92}{17\cdot 6 \times 17\cdot 1}\sqrt{g\,H} = 1\cdot 43 \sqrt{g\,H}$$

greater than w_1, so that there appears to be an increase of velocity in the volute. Probably the real orifice is about ·85 that given by the anemometer, and w_1 is in reality much larger, and v_4 less. It is, however, clear that a chimney is necessary to reduce v_4, which would cause a loss of head, if not reduced, of

$$\frac{v_4^2}{2g} = 1\cdot 02\,H,$$

so that the efficiency would be below 50 per cent.

CHAPTER XII.

28. *Comments on the preceding Experiments, and Comparison between Theory and the Results of Experiments.*—One of the most important statements that we made was that the general equation of any fan could be thrown into the form

$$c_1^2 + P c_1 Q - R Q^2 - S g H = o \quad . \quad . \quad . \quad (22)$$

and that this could be put into the form

$$\frac{1}{M} + \frac{P.O.}{\sqrt{M}} - R O^2 - S = o \quad . \quad . \quad . \quad . \quad (39)$$

The following table gives the calculated and experimental values for H for the Rateau fan of Table XII. The quantities are in this case in the metric system. The equation of the fan is

$$c_1^3 + 2{\cdot}64 c_1 Q - 1{\cdot}91 Q^2 - 19{\cdot}62 H = o.$$

No. of equipment.	c_1 in metres.	Q per sec. in cubic metres.	H: Actual.	H: Calculated from the above equation.
1	29·4	35·3	63	62·5
2	29·4	35	65	64·2
3	30·4	34·2	74	71
4	32	32·7	88	89
5	31·35	27·6	92	92·5
6	34·1	23·4	113	113·5
7	34·2	16·5	110	108
8	32·8	7·8	84	83·4
9	29·2	1·3	44	47
10	34·4	18·5	113	112

The last two columns show how very closely the fan obeys the law.

The next table gives a comparison of the manometric efficiency obtained by experiment and that from

$$\frac{1}{M} - {\cdot}136 \frac{O}{\sqrt{M}} - 14{\cdot}18 O^2 - 1{\cdot}69 = 0,$$

which is equation (39) for fan No. VIII. of Mr. Bryan Donkin's experiments, and fans VI., X., and XI. have been treated in a similar manner.

Equivalent orifice in square feet.	Actual manometric efficiency. Per cent.	Calculated manometric efficiency. Per cent.
0	59·0	59·0
0·1	52·5	54·5
0·2	43·0	43·5
0·3	32·0	33·1
0·4	24·0	24·7
0·5	19·0	18·6
0·8	9·5	9·0
1·0	6·6	6·1
1·5	3·0	2·87

FAN NO. VI. MR. BRYAN DONKIN'S EXPERIMENTS.

Equivalent orifice in square feet.	Calculated. manometric efficiency. Per cent.	Actual manometric efficiency. Per cent.
0	60·0	60·0
0·1	57·5	58·0
0·2	54·1	54·0
0·3	50·0	50·0
0·4	40·4	43·0
0·5	36·25	36·25
0·8	20·7	22·0
1·0	14·25	15·0

The equation to the above fan is—

$$\frac{1}{\mu} + \cdot 192 \frac{O}{\sqrt{\mu}} - 5\cdot 85\, O^2 - 1\cdot 666 = 0.$$

FAN NO. X. MR. BRYAN DONKIN'S EXPERIMENTS.

Equivalent orifice in square feet.	Calculated manometric efficiency. Per cent.	Actual manometric efficiency. Per cent.
0	57·0	57·0
0·1	63·0	60·0
0·2	59·8	59·8
0·3	51·0	52·0
0·4	41·6	41·6
0·5	33·0	33·0
0·8	17·7	17·85
1·0	12·1	16·0
1·5	6·34	7·5

The equation to the above fan is—

$$\frac{1}{\mu} + 2{\cdot}43 \frac{O}{\sqrt{\mu}} - 13{\cdot}55\,O^2 - 1{\cdot}755 = 0.$$

FAN NO. XI. MR. BRYAN DONKIN'S EXPERIMENTS.

Equivalent crifice in square feet.	Calculated manometric efficiency. Per cent.	Actual manometric efficiency. Per cent.
0	28·5	28·5
0 1*	32·8	26·0
0·24	19·0	19·0
0·3	14·5	15·5
0·4	9·47	10·5
0·5	6·5	6·5
0·8	2·78	2·8
1·0	1·8	2·0
1·5	·08	1·0

The equation to the above fan is—

$$\frac{1}{\mu} + 10{\cdot}05 \frac{O}{\sqrt{\mu}} - 126{\cdot}4\,O^2 - 3{\cdot}51 = 0.$$

Fan No. 1, in the same paper, has an equation

$$c_1^2 + {\cdot}0649\, c_1 Q - 4440\, h - {\cdot}00196\, Q^2 = 0,$$

where c_1 is the peripheral velocity in feet per second, and Q is the number of cubic feet of air per minute, h being the static water gauge. This, however, is not so good a proof of (22) as the preceding, as the static water gauges were often in Mr. Donkin's experiments lower than they should have been owing to induction, and a vacuum was often shown where there was undoubtedly pressure, for how could the air flowing in a pipe of uniform section pass through baffle plates, and yet with unchanged velocity reach a space of greater pressure, a manifest contradiction of the law of the conservation of energy. The reason for the apparent vacuum was that induction took place, as explained in Messrs. Heenan and Gilbert's paper, and which they avoided by using the tip in fig. 38. We believe that owing to this induction the water gauge is too great in many fan experi-

* The smallest equivalent orifice at which a test was made, except zero orifice, was ·24 square feet.

ments. The following table gives the static water gauges for fan No. 1, obtained by experiment, and also the values of h calculated from the above equation.

Actual static water gauge :—	$\frac{3}{4}$ vac.	$\frac{1}{2}$ vac.	$\frac{1}{8}$ vac.	$1\frac{1}{8}$ pres.	$2\frac{1}{2}$	$3\frac{7}{8}$	$5\frac{5}{8}$	$5\frac{3}{4}$	$5\frac{1}{4}$	3[illegible]
h :—	·61 vac.	$\frac{1}{2}$ vac.	·1745 pres.	1·09 pres.	2·52	3·94	$5\frac{5}{8}$	5·81	5·07	3·87
Difference :—	− ·14	0	− ·2295	+ ·035	+ ·02	− ·065	·0	− 006	+ ·18	− ·058

Table of Manometric Efficiencies of the Parkend Mine Fan.

Volume in cubic feet per minute per square inch of diametrical section, $\div \sqrt{\text{water gauge}}$.	Manometric efficiency.	Tip speed in feet per minute.
Zero	·604	9000
Zero	·602	5000
Zero	·600	8000
Zero	·603	6000
·91	·6 nearly	8000
1·48	·603	6000
2·76	·587	8000
3·68	·594	9000
4·51	·563	5000
5·06	·559	6000
5·68	·562	8000
5·89	·565	9000
5·91	·554	9000
7·34	·483	5000
7·62	·495	9000
8·48	·447	6000
10·39	·375	8000
12	·322	5000
12·10	·327	9000
15·5	·237	9000
16·0	·224	6000
23·5	·125	8000
26·8	·100	8000
27·2	·099	9000

The Parkend Mine fan, tested by Messrs. Heenan and Gilbert, has manometric efficiencies that do not vary so long as the orifice remains constant. The preceding table contains four experiments at a tip speed of 5,000 ft. per minute, five at 6,000, seven at 8,000, and eight at 9,000. The first column is proportional to the equivalent orifice; the manometric efficiency has been calculated from the equation

$$M = \frac{10000}{144}\frac{g.h.}{c_1^2}.$$

Although this is not direct support of (22), it shows that, for a fixed value of $\frac{Q}{\sqrt{g}\,H}$, $\frac{c_1}{\sqrt{g}\,H}$ is constant, and therefore $\frac{c_1}{Q}$ is constant.

We now come to a formula that is much harder to prove, viz., that the air efficiency

$$\eta = \frac{g\,H}{c_1 \,.\, w_1}.$$

The difficulty is due to the exaggeration of the discharge and possibly also of the water gauge. It is also in many cases difficult to determine ϕ, and therefore w_1, and in many cases ϕ is a variable quantity. In Mr. Donkin's paper, however, where the discharge may be relied on, there are two fans in which the agreement is almost perfect except at very small orifices, where the formula does not hold good, owing to the fact that losses of energy which we neglect in the above formula become of consequence. At these small orifices the friction between the wheel disc and the air, which varies at c^2, and the work due to it as c_1^3, or the friction of the air against the fan casing during its passage through the wheel, which is a quantity of the second degree in c_1 and Q—*i.e.*, it may be expressed by the formula

$$c_1^2 + m\,c_1\,Q + n\,Q^2,$$

become of importance. This will be clear to the reader when we point out that when Q is zero

$$\frac{g\,H}{c_1\,w_1} = \frac{g\,H}{c_1^2} = M = \tfrac{1}{2} \text{ on the average,}$$

but as the efficiency is zero at zero orifice, η must differ considerably from it.

Fan No. 1 in Mr. Donkin's experiments is a Rateau fan, and ten experiments with this are given in Table II. There are twenty wrought-iron vanes which are inclined forwards at 45 deg. to the tangent to the outer circumference. Also from Table I.

$$u_1 = \frac{Q}{2\pi r_1 b_1} = \frac{Q}{r_1^2}.$$

Where Q = cubic feet per second,

$$w_1 = c_1 - u_1 \cot \phi = c_1 + u_1,$$

as $\phi = 135$ deg., if we assume that the angle of discharge coincides with the vane angle, and that there is no coefficient of contraction at discharge from the wheel, both of which are extremely probable as there are twenty vanes (a large number for a diameter of 19·6 in.), and the vanes are so designed that uniform outflow probably takes place. It has been proved by experiment that the inflow to the fan is uniform. Then

$$\frac{gH}{c_1 w_1} = \frac{32{\cdot}2 \times 10000\, h}{144\, c_1 (c_1 + u_1)}$$

where h is the dynamic water gauge. As an example in experiment 6,

$$Q = 2700, \qquad c_1 = 110{\cdot}8,$$

$$u_1 = 67{\cdot}2, \qquad w_1 = 178,$$

$$h = 4{\cdot}75\,; \qquad \therefore \quad \frac{gH}{c_1 w_1} = {\cdot}54,$$

as compared with ·599 in the experiment.

The following table gives the experimental efficiencies and the corresponding values of η:—

Experimental efficiency....	12	17·1	21·3	33·9	47	59·4	59·9	60·6	49·9	0
Calculated air efficiency	12	15·25	20·85	31·7	44	59·8	54	57·75	62·5	50
Air efficiency difference ..	0	1·75	·15	·2	3	−·1	5·9	2·85	−12·6	−50

so that the agreement is very close except in the last two cases, which is to be expected for the reasons already given.

Fan No. 3 had plane radial vanes, so that the outflow must have been radial, and whether uniform or no is of no consequence, because $c_1 = w_1$. It will be clear to the reader that the outflow must have been radial, because we are justified in supposing the wheel fixed and a centrifugal accelerative force acting on each particle. It must not be imagined that all fans whose vanes have radial tips have radial outflow. Thus blade No. 4, fig. 17, probably causes a backward relative outflow, owing to the fact that the vanes at first curve backwards from the inner radius. In fan No. 3 we should therefore expect that the manometric efficiency and air efficiency would be the same, except at small orifices, and this we find is the case. By measurement from figs. 44 and 45, we have—

Equivalent orifice.........	·2	·4	·6	·8	1·0	1·2	1·4	1·6
Manometric efficiency	67·5	61·5	55·0	46	38·0	32·5	27·5	24·5
Air efficiency..............	57·5	63	57·5	47·5	40·0	33·5	30	26·0

Except in the first case, the difference is never more than $2\frac{1}{2}$ per cent.

In all experiments with the Rateau fan, in which the anemometer was used, we find η smaller than it should be, and most markedly so in those experiments by the Belgian Commission, in which the efficiencies were unusually high. In all these experiments we can calculate η thus—

$$\eta = \frac{g\,H}{c_1\,w_1} = \frac{\sqrt{g}\,H\ \sqrt{M}}{c_1 + u_1} = \frac{\sqrt{M}}{\dfrac{1}{\sqrt{M}} + O_1}$$

$$= \frac{M}{1 + O_1\,\sqrt{M}} \text{ where } O_1 \text{ is the reduced orifice } \frac{Q}{r_1^2\,\sqrt{g}\,H}.$$

From Table XII. we get the following results—

No. of experiment	1	2	3	4	5	6	7	8	10
η per cent	32·3	33·85	37·4	41·7	48·5	56·4	62·7	60·7	61·2
Mechanical efficiency per cent.........	33	37	43	46	53	60	61	43	59

From Table XIV.—

No. of experiment........	1	5	6	7	8	9
η per cent.	57	47	32·3	54	52·2	51·6
Mechanical efficiency per cent.	56	48	35	57	57	53

From Table VIII., taking the third experiment with each fan—

No. of fan.....................	1	2	3	4
η per cent......................	29·8	44·4	54·5	50·5
Mechanical efficiency per cent..	48·3	82·6	47·0	77·5

In all the above except 7, 8, 10 of XII., 1 of XIV., and 3 VIII., the values of η are less than the mechanical efficiencies, although these include engine friction. If we multiply η by $\frac{9}{10}$, which is the highest engine mechanical efficiency we are justified in assuming, all except 3 of VIII., and 8 of XII., which is at a very small reduced orifice, are below the values in the third lines, and the greatest discrepancies are those in VIII., where the experimenters acknowledge that their discharges are too high. It is reasonable to suppose that the discrepancies are therefore mainly due to exaggerated discharge, because in this case it would reduce η, for by an increase of u_1 we increase w_1. In the following table we have supposed the actual discharge in Table XII. to be $\frac{9}{10}$ths of that given by the anemometer, so that η is increased and the mechanical efficiency reduced. We get the following results :—

No. of experiment.	1	2	3	4	5	6	7	8	10
η per cent	34·2	35·8	39·5	44·1	50·9	58 9	59·0	62	63·5
Mech. effic. per cent	29·7	33·3	38·6	41·4	47·6	54·0	54·9	38·7	53·1
$\frac{\text{Mechan'l efficiency}}{\eta}$	·87	·93	·97	·94	·935	·916	·93	·625	·835

Now, except at small orifices such as 8 and 9, which is omitted, as it is practically zero,

Efficiency of fan $= \eta$ very nearly

$$= \frac{\text{efficiency of engine and fan}}{\text{efficiency of engine}}.$$

Hence efficiency of engine

$$= \frac{\text{efficiency of engine and fan}}{\eta},$$

and should be about ·85 to ·90. We infer from the above that the exaggeration of the discharge is greater than what we have assumed, viz., the ratio $\frac{10}{9}$, but not very much, and this agrees fairly well with the results of the Prussian Commission if we allow for the additional exaggeration due to the variable velocity of the air which is always found in mines.

CHAPTER XIII.

Let us next consider Messrs. Heenan and Gilbert's experiment with a fan 17 in. diameter and 8 in. wide. The efficiency here given is the ratio of the useful work done by the fan to the work done on a shaft which drives the fan by a belt, so that it should not be much less than the efficiency of the fan alone, or η at orifices of moderate size, if we can assume the truth of the statement that the work done per pound by the wheel is $c_1\, w_1 \div g$. In fig. 18 we have the results of experiments with a blade terminating at 35 deg. to the circumference, so that if the angle of relative outflow were the same as the vane angle we should have $\phi = 35$ deg. Consider the experiment in which the discharge was 2,000 cubic feet per minute, and the tip speed 12,000 ft. per minute. The total water gauge is 9·6 in., so that we have

$$c_1 = \frac{12000}{60} = 200,\ Q = \frac{2000}{60},\ \text{and } u_1 = \frac{Q}{2\,\pi\, r_1\, b_1}$$

$$= \frac{2000 \times 144}{60 \times \pi \times 17 \times 8} = 11{\cdot}22 \text{ ft. per sec.}$$

$\eta = {\cdot}635$ from the diagram of total efficiency, and

$$H = 9{\cdot}6 \times \frac{10000}{134} \text{ ft.}$$

$$\therefore\ w_1 = \frac{g\,H}{c_1\,\eta} = \frac{32{\cdot}2 \times 96000}{200 \times 144 \times {\cdot}635} = 169.$$

$$\cot\phi = \frac{c_1 - w_1}{u_1} = \frac{31}{11{\cdot}22} = 2{\cdot}76,\ \phi = 19^\circ\ 55'.$$

Now, with a vane of this type the relative angle of outflow cannot be less than the angle of vane, although if there were double curvature, as with blade No. 4, this might very well be the case. Hence we may reasonably assume here that ϕ should be greater, η consequently less, and w_1 more; so that cot ϕ is less. If we even assume such a small coefficient of contraction as ·8, so that

$$u_1 = \frac{11 \cdot 22}{\cdot 8}, \text{ and } \cot \phi = 2 \cdot 2,$$

$\phi = 24° \, 26'$, which is still too small.

We may here mention that in radial-flow turbines it is customary to assume a coefficient of contraction, and the value of ·9 gives the best agreement between theory and practice. The obvious deduction is that the discharge is exaggerated, and it is stated by Prof. Rateau in the discussion on this paper that as the delivery of the air had been measured in a section presenting an abrupt contraction of the tube, and as the contraction of the filaments of air caused an irregularity in the record of the anemometer, that there would be an exaggeration in addition to that found by the Prussian Commission, where the flow of air was uniform. He was led to that conclusion by the results of many experiments he had conducted on fans.

If we consider the discharge of 3,500 cubic feet per minute

$$Q = \frac{3500}{60}, \quad u_1 = \frac{3500 \times 144}{60 \times \pi \times 17 \times 8} = 19 \cdot 7,$$

$c_1 = 200$, as before, and $\eta = \cdot 7$ dynamic or total efficiency,

$$H = 7 \cdot 6 \times \frac{10000}{144} \text{ ft.}, \quad w_1 = \frac{32 \cdot 2 \times 76000}{200 \times 144 \times \cdot 7} = 121,$$

$$\cot \phi = \frac{200 - 121}{19 \cdot 7} = \frac{79}{19 \cdot 7} = 4 \cdot 01, \quad \phi = 14 \text{ deg.},$$

and even if we suppose a coefficient of contraction of ·8,

$$\phi = 17 \text{ deg. } 18 \text{ min.}$$

Fig. 20 shows similar curves for blade No. 4, fig. 17, which has double curvature, and terminates radially at the outer circumference, so that if the angle of flow was the same as the angle of vanes, ϕ would be 90 deg. The dynamic efficiency, when the flow is 2,500 cubic feet per minute, is 83 per cent, and the dynamic gauge is 12·8 in.,

$$u_1 = \frac{2500 \times 144}{60 \times \pi \times 17 \times 8} = 14{\cdot}05,$$

$$H = \frac{128000}{144} = 890, \qquad w_1 = \frac{32{\cdot}2 \times 890}{200 \times {\cdot}83} = 172,$$

$$\cot \phi = \frac{200 - 172}{14{\cdot}5} = \frac{28}{14{\cdot}5} = 1{\cdot}93,$$

$$\phi = 27 \text{ deg. } 23 \text{ min.},$$

which we consider extremely improbable, as there are six vanes. In our opinion the discharge is very much exaggerated. The manometric efficiency in this case is

$$M = \frac{g\,H}{c_1^2} = \frac{32{\cdot}2 \times 890}{200 \times 200} = 71{\cdot}6 \text{ per cent},$$

which is probably very much nearer the true value of the mechanical efficiency. At the same discharge the compression or static gauge is 11 in., and this gives

$$w_1 = \frac{32{\cdot}2 \times 110000}{200 \times 144 \times {\cdot}7} = 175{\cdot}5$$

since the mechanical efficiency is 70 per cent;

$$\cot \phi = \frac{200 - 175{\cdot}5}{14{\cdot}5} = \frac{24{\cdot}5}{14{\cdot}5} = 1{\cdot}69,$$

$$\phi = 30 \text{ deg. } 36 \text{ min.},$$

while the manometric efficiency is

$$M = \frac{32{\cdot}2 \times 110000}{200^2 \times 144} = {\cdot}615.$$

The discharge of 4,000 cubic feet per minute gives the following results: $u_1 = 22{\cdot}5$, $w_1 = 152$, $\cot \phi = 2{\cdot}13$, $\phi = 28$ deg., the dynamic efficiency being ·85 and the dynamic water gauge 11·6 in., while the manometric efficiency is 64·7 per cent. We do not think that the mechanical efficiency is as low as the manometric efficiency, because it is quite possible that ϕ may be less than 90 deg., and we believe that it is. In support of this statement we mention some experiments with a Farcot centrifugal pump at Khatatbeh, Egypt,* which were made with very great care, and are probably accurate. In these the mechanical efficiency of engine and pumps was 65 per cent, corresponding to a probable efficiency of pump alone of between 72·2 and 76·5 per cent, while the manometric efficiency was 65·9 per cent.

We shall now discuss the experiment with the Seghill Colliery open-running fan. Before doing so we may state that we believe the efficiency is exaggerated. Not knowing the section of the fan drift, we cannot find the correct reduction of the water gauge due to the velocity of the air therein; but, neglecting this, let us consider the total energy of air rejected from the outer circumference of the fan. In the second average of experiments

$$2\,r_1 = 35{\cdot}08 \text{ ft.}, \qquad b_1 = 1{\cdot}292,$$

and the cubic feet of air per second

$$Q = \frac{135742}{60} = 2265 \text{ cubic feet},$$

$$u_1 = \frac{2265}{\pi \times 35{\cdot}08 \times 1{\cdot}292}, \text{ neglecting contraction,}$$

which is favouring the fan, because it reduces the rejected energy;

$$w_1 = \frac{g\,.\,H}{c_1\,.\,\eta},$$

* Annales des Poutes et Chausées, 1888.

and w_1 will be least if we give η its largest possible value. Assume that the mechanical efficiency of the engine was only 85 per cent, and we get

$$w_1 = \frac{32 \times 24600 \times 85}{144 \times 111 \times 54 \cdot 78} = 77,$$

and assuming the density of the air as ·0761, the energy rejected at outflow in H.P. is

$$\frac{u_1^2 + w_1^2}{2g} \times Q \times \frac{\cdot 0761}{550} = \frac{77^2 + 15 \cdot 9^2}{64} \times \frac{2265 \times \cdot 0761}{550} = 30 \cdot 2.$$

Adding this to 52·65 useful H.P. we account for 82·85 H.P. of the 96·114 I.H.P., giving a total dynamic efficiency of ·862 after having assumed that the efficiency of the engine alone is ·85, which is of course impossible. Treating experiment 18 in the same way we get $u_1 = 29$, $c_1 = 110$, $w_1 = 67 \cdot 5$, and H.P. rejected = 49·2, so that the dynamic efficiency is

$$\frac{83 \cdot 47 + 49 \cdot 2}{153 \cdot 67} = \cdot 864.$$

Next let us suppose that the engine efficiency is higher, say 90 per cent; in the first of the above two cases $w_1 = 81 \cdot 5$, and the H.P. rejected

$$= \frac{(81 \cdot 5)^2 + (15 \cdot 9)^2}{64} \times \frac{2265 \times \cdot 0761}{550} = 33 \cdot 7,$$

so that the dynamic efficiency of engine and fan is

$$\frac{52 \cdot 65 + 33 \cdot 7}{96 \cdot 11} = \cdot 90, \text{ very nearly,}$$

allowing next to nothing for the losses by friction in the fan. In experiment 18 we should have $w_1 = 71 \cdot 4$, and the H.P. rejected is

$$= \frac{(71 \cdot 4)^2 + (29)^2}{64} \times \frac{247520}{33000} \times \cdot 0761 = 54\ 3,$$

so that the dynamic efficiency is

$$\frac{83 \cdot 47 + 54 \cdot 3}{153 \cdot 67} = \cdot 895.$$

These experiments are therefore conclusive proofs of the exaggeration of either water gauge or anemometer. They are also an illustration of the absurdity of estimating the excellence of a fan by its dynamic efficiency.

Consider next the experiments with the Waddle fan, section 25. In the first place we do not believe that the expanding rim has the desired effect. Foreign engineers seem to consider an angle of inclination of 7 deg. to be sufficient for the sides of a diffuser, and we do not think that air travelling at such a high velocity could accommodate itself to such a rapid curve, but in what follows we shall suppose that it does, and that

$$u_3 = \frac{Q}{2 \pi r_3 . b_3} = \frac{231306}{60 \times \pi \times 36\frac{1}{3} \times 2 \cdot 23} = 15 \cdot 2 \text{ ft. per sec}$$

radial component of velocity of flow from the diffuser in experiment 4;

$$w_3 = \text{tangential component} = w_1 \frac{r_1}{r_3}$$

$$= \frac{g . H . r_1}{c_1 . \eta . r_3} = \frac{32 \times 26710 \times 35}{109 \cdot 2 \times 144 \times \cdot 792 \times 36\frac{1}{3}} = 66 \cdot 2$$

assuming a mechanical efficiency of 85 per cent for the engine alone; H.P. rejected in the form of kinetic energy

$$= \frac{w_3^2 + u_3^2}{2g} \times \frac{Q \times \cdot 0761}{550} = 38 \cdot 6.$$

Adding to this useful H.P. 97·33 we find the dynamic efficiency is

$$\frac{38 \cdot 6 + 97 \cdot 33}{144 \cdot 5} = 94 \cdot 1 \text{ per cent,}$$

which is, of course, impossible. If we assume that the mechanical efficiency of the engine alone is 90 per cent, the dynamic efficiency becomes

$$\frac{42\cdot9 + 97\cdot33}{144\cdot5} = 97 \text{ per cent.}$$

The only conclusion is that the anemometer very greatly exaggerates the discharge of air.

CHAPTER XIV.

High-pressure Fans.—The following is a summary of a paper by Prof. A. Rateau, published in the "Bulletin de la Société de l'Industrie Minerale":—

Hitherto fans have not been required to give a water gauge of more than 24 in., while it is only lately that centrifugal pumps have been used for heads over 50 ft. By means of steam turbines and a single pump a head of nearly 1,000 ft. has been obtained, while with several wheels placed in series electro-motors can be used. These turbines, pumps, and fans are the design of Prof. Rateau, and are constructed by Messrs. Sautter-Harlé, of Paris.

In the theory of centrifugal pumps and fans there are four quantities of importance. The mechanical efficiency

$$\rho = \frac{D\,Q\,H}{T_m}.$$

The volumetric efficiency

$$V = \frac{Q}{c_1\,r_1^2}.$$

The manometric efficiency

$$M = \frac{g\,H}{c_1^2}.$$

The coefficient of power transmitted to the shaft of the pump

$$\tau = \frac{g\,T_m}{c_1^3\,r_1^3} = \frac{M\,V}{\rho}.$$

The above are numbers independent of all units. In the formulæ Q is the volume in cubic feet or cubic metres per second, H is the head in metres or feet, D is the weight in pounds or kilogrammes of 1 cubic foot or metre of the fluid pumped, r_1 is the external radius of the wheel, T_m is the work done per second in foot-pounds or kilogrammetres, and c_1 is the velocity of the wheel at the outer radius. Prof. Rateau used δ as abscissæ instead of orifices

$$\frac{Q}{\sqrt{2gH}}$$

and draws three curves of M, ρ, and τ, as in figs. 86 and 87, the former being those of a centrifugal pump, while the latter is that of a Rateau fan. For a given pump at a fixed number of revolutions per minute, the curves τ and M are those of T_m and H to suitable scales.

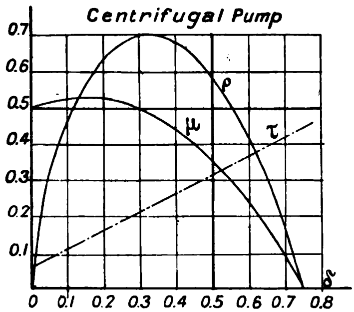


FIG. 86.

Figs. 88, 89, 90 show a centrifugal fan driven by a steam turbine and intended to produce a considerable pressure. The fan is made of steel of very good quality, capable of running at a peripheral velocity of over 800 ft. per second; it turns in a cast-iron casing having two openings for suction and forming a diffuser and volute. The turbine, which is a steam Pelton wheel, is 11·8 in. in diameter, while the fan is 10 in.; the method of raising the oil from a lower to a higher reservoir is shown in fig. 91, in which the pipe A is connected to the lower reservoir and B to the higher. The small tube *m* brings a small amount of air from the fan, which, mixing in small bubbles with the column of oil B, lowers its specific gravity to an extent sufficient to enable the column A to raise it to the higher reservoir. The discharge or pressure of the fan can be controlled by a pneumatic governor. It consists of a cylinder D, fig. 88, containing a piston, the details of which are shown in I,

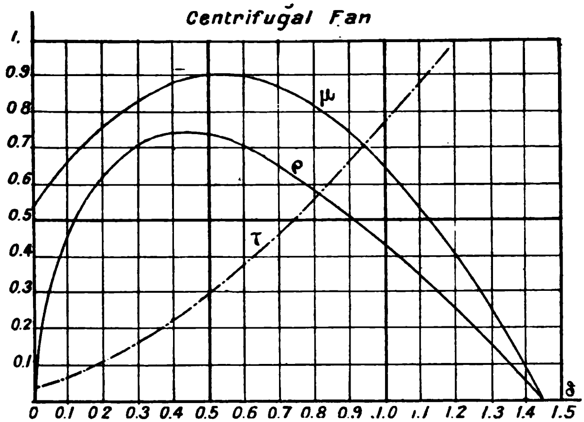


Fig. 87.

fig. 93. The rod of this piston is connected by a short rod to the point *c* of the lever *a b c*, which oscillates about the point *b*, and is connected to the rod of the throttle valve at *a*. The cylinder D has its two ends connected by the pipes *e*, *f*, figs. 88 and 89, with a straight tube *e* and a Pitot tube *f*, both placed in the discharge pipe of the fan; see also H and G, fig. 93 Thus the difference of the pressures on the two sides of the piston D is proportional to the square of the velocity of discharge, and exerts an upward pressure which is balanced by the weight of the piston, assisted by an additional weight if necessary and a spring *r*, fig. 88, whose tension increases with the rise of the piston and

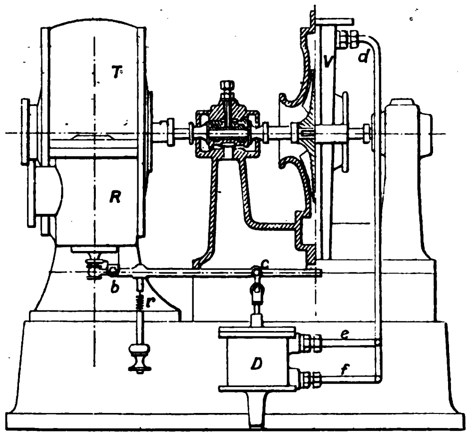


FIG. 88.

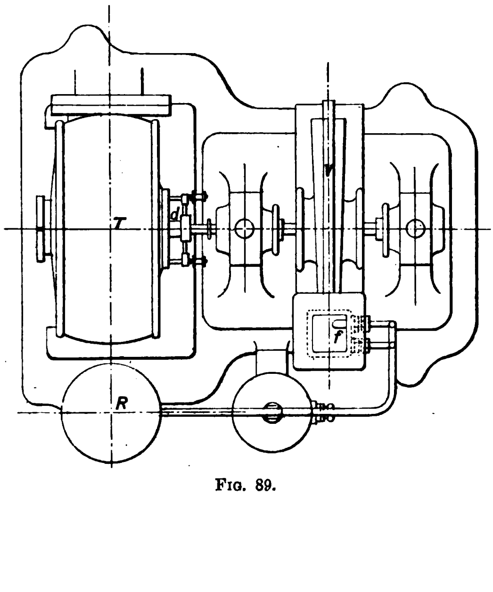


FIG. 89.

gives stability. If a constant pressure instead of volume is required, the upper part of the cylinder is connected to the atmosphere and the lower is left as above. To avoid piston friction a caout-chouc diaphragm is sometimes used. The experiments were made at Messrs. Sautter-Harlé's Paris

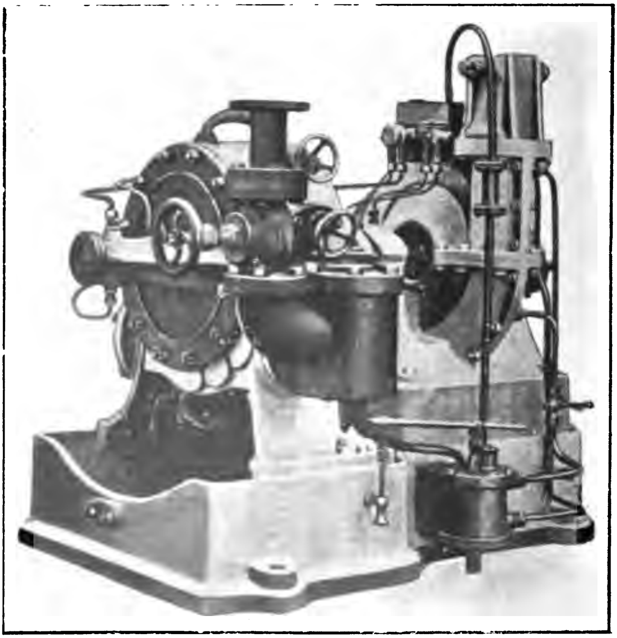


FIG 90.

works, and the measurements by M. Rateau and M. Chatelain. The discharge was measured by means of a convergent discharge pipe, two of whose faces were fixed and parallel, while the other two were movable, like a duck's beak. At its larger end this was fixed to the discharge pipe of the fan, and it was thus possible to vary the discharge;

a mercury manometer was used to measure the pressure in this convergent mouthpiece, and the discharge could be calculated from·this. The speed was changed from 8,000 to 20,200 revolutions per minute. The peripheral speed of the fan nearly reached 870 ft. per second, while the discharge pressure amounted to 16·75 in. of mercury, or 228 of water—more than half an atmosphere. The revolutions were measured by a counter driven by worm gear, which reduced the speed one-thirtieth. One, two, or three of the steam turbine nozzles were opened to give the power required by the fan, and the steam pressure P in the steam

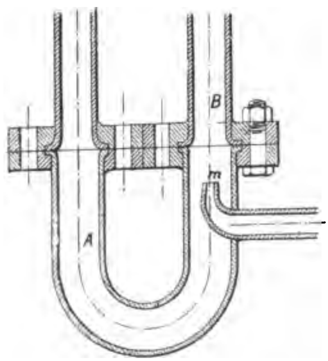


FIG. 91.

chest was noted, as by this means the power of the turbine could be deduced from previous experiments, so that the total efficiency of engine and fan was thus obtained.

By formulæ* obtained by M. Rateau the consumption of steam, and consequently the power theoretically available, could be calculated. The governor was not used during these experiments, so that the pressure in the steam chest was not affected by it. The discharge Q, the useful power T_u, the theoretical power of the steam T_t, and the coefficients M, ρ, and V were calculated in the following manner: The

* See Rapport au Congrès International de Mécanique Applique.

discharge Q, in cubic feet per second, estimated at atmospheric pressure, is equal to the product of S, the section of the convergent discharge pipe in square feet, and the velocity of flow v in feet per second. Hirn's experiments prove that the coefficient of discharge of a convergent pipe does not differ more than 1 or 2 per cent from unity. M. Rateau proves in his paper that without serious error

$$\frac{v^2}{2g} = \frac{\Delta p}{D},$$

where Δp is the difference of pressure per square foot between suction and discharge, and D is the mean density of the air. Of course, the same formula would hold good with the metre as the unit of length. The figures in the seventh column of the following table are obtained in this manner. The sixth column gives H in feet of water, and in the calculation of D the temperature must be assumed to be 38 deg. Cen. The reduction of volume during compression must be taken into account in calculating the useful power T_u. To obtain the figures in the tenth column the variation of pressure per square foot must be multiplied by the volume in cubic feet at the mean pressure, or

$$Q \frac{32{\cdot}8}{32{\cdot}8 + 0{\cdot}5\,H}.$$

The work theoretically obtainable from the steam T_t is calculated as follows: The discharge of steam is obtained from the formula—

$$I = s\,P\,(15{\cdot}20 - 0{\cdot}96 \log P),$$

where s is the total section of the nozzles in square centimetres, and P is the pressure in kilogrammes per square centimetre, while I is the quantity of steam discharged per second in grammes; if this is multiplied by 3·6, it gives kilogrammes per hour. If K is the number of kilogrammes per horse power hour, when the steam chest pressure is P and the exhaust p, in this case the atmospheric pressure, then

$$K = 0{\cdot}85 + \frac{6{\cdot}95 - 0{\cdot}92 \log P}{\log P - \log p}.$$

High-pressure Turbine Fans.

Diameters of turbine and fan, 10 in.; steam nozzles, ·355 in. diameter; date of experiments, July 24th and 25th, 1900.

Number of experiment.	Section of discharge orifice in square inches.	Number of steam nozzles open.	Steam chest pressure in pounds per square inch. P.	Revolutions per minute. N.	Water gauge in feet. H.	Cubic feet of air discharged per second. Q.	$\frac{10^8 H}{N^2}$. H in metres.	Volumetric efficiency. V.	Useful work in horse power. T_u.	Work theoretically obtainable from the steam in horse power. T_t.	Efficiency of turbine and fan. $\rho_o = \frac{T_u}{T_t}$
1	1·28	2	48·3	10500	4·56	1·875	1·26	0·069	2·22	18·2	0·122
2	1·28	2	62·5	13000	6·82	1·810	1·23	0·063	4·25	28·2	0·151
3	1·28	2	76·6	15100	9·35	1·915	1·25	0·058	6·01	39·0	0·154
4	1·28	2	90·9	16950	11·97	2·31	1·27	0·062	9·05	50·3	0·180
5	1·28	2	105	18500	14·75	2·30	1·32	0·057	10·85	62·4	0·174
6	2·39	2	38·3	8200	2·76	2·03	1·25	0·112	2·03	11·5	0·177
7	2·39	2	40·5	10000	4·52	2·57	1·38	0·117	4·13	22·5	0·184
8	2·39	2	78·0	13700	8·26	3·39	1·35	0·113	9·52	41·1	0·232
9	2·39	2	94·6	15400	10·6	3·78	1·36	0·112	13·28	53·7	0·229
10	2·39	2	105·0	16600	12·8	4·02	1·36	0·110	16·16	62·4	0·259
11	2·39	2	118·0	17700	14·2	4·28	1·38	0·110	19·38	73·9	0·262
12	3·54	2	49·6	9350	3·94	3·57	1·37	0·174	5·02	19·2	0·261
13	3·54	2	76·6	12050	6·26	4·44	1·32	0·167	9·62	39·0	0·247
14	3·54	2	103·5	14750	9·51	5·34	1·33	0·165	17·04	61·1	0·279
15	3·54	2	119·2	15900	10·9	5·58	1·32	0·160	20·12	75·2	0·268
16	3·54	2	133·2	16950	12·7	6·05	1·34	0·162	25·67	88·2	0·291
17	4·7	2	62·5	9500	3·51	4·51	1·18	0·217	5·74	28·2	0·204
18	4·7	2	90·7	12500	6·03	5·80	1·18	0·211	12·20	50·3	0·243
19	4·7	2	119·2	14800	8·86	6·87	1·23	0·211	20·53	75·2	0·273
20	4·7	2	143·2	16500	11·27	7·64	1·26	0·212	28·36	97·3	0·292
21	2·95	3	85·1	15400	10·35	4·61	1·34	0·136	15·93	68·5	0·233
22	2·95	3	106·5	17750	13·95	5·23	1·37	0·134	23·48	95·5	0·246
23	2·95	3	132	20200	19·0	5·92	1·42	0·134	34·75	130·0	0·267
24	3·54	3	90·7	15200	10·1	5·47	1·33	0·164	18·49	76·0	0·243
25	3·54	3	116·2	17750	13·75	6·25	1·34	0·160	27·71	109	0·254
26	3·54	3	147·5	20200	18·70	7·05	1·40	0·159	40·72	152	0·307
27	4·12	3	95	15400	9·7	6·29	1·25	0·185	20·43	81	0·252
28	4·12	3	125	17750	13·3	7·19	1·30	0·184	32·01	120	0·267
29	4·12	3	155	20200	18·05	8·12	1·35	0·183	45·55	162	0·281

Then the theoretical horse power obtainable from the steam is

$$T_t = \frac{I}{K}.$$

The mechanical efficiency

$$\rho = \frac{T_u}{T_t},$$

and is given in the last column of the table on page 189. As the density of the air varies at different speeds, owing to

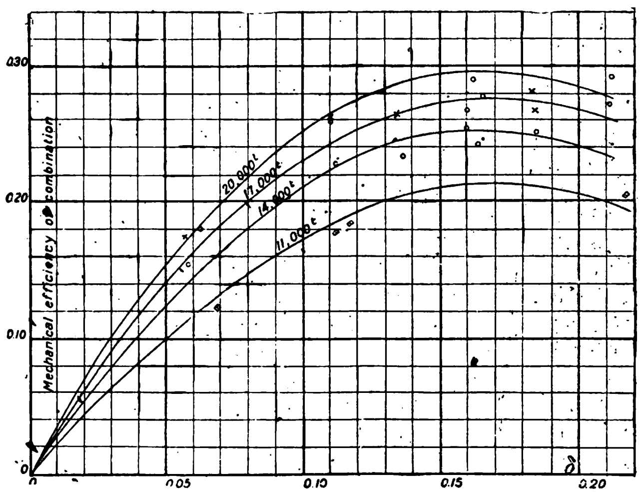


FIG. 92.

the great compression, the manometric efficiency is not constant for a given opening of the convergent discharge pipe. In its place in the eighth column we give

$$\frac{10^8 H}{N^2}$$

where N is the number of revolutions per minute; it lies between 1·18 and 1·42 when H is in metres. It is, however, not a constant for a given opening of the discharge pipe, but increases with N. Although the fan is only 10 in. diameter

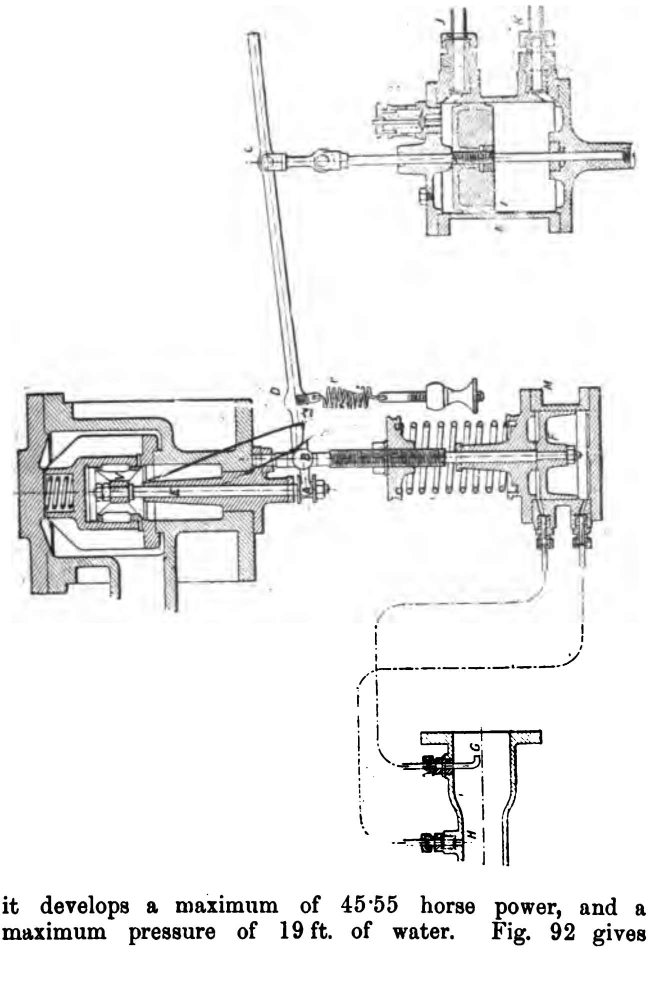


FIG. 93.

it develops a maximum of 45·55 horse power, and a maximum pressure of 19 ft. of water. Fig. 92 gives characteristic curves, a separate characteristic being given for each speed as the efficiency of the turbine increases with the speed. The ordinates are values of ρ, while the abscissæ are values of V. *It must be remembered that the efficiency is the ratio of the useful work done by the fan to the ideal amount obtainable from the steam in the Rankine or Clausius cycle.* At normal speed the efficiency passes 28 per cent, showing that the fan alone has an efficiency of 56 per cent, as that of the turbine is about 50 per cent, according to experiments previously made.

High-pressure fans driven by steam turbines can be used for cupolas, blast furnaces, and Bessemer converters, and wherever a water gauge of more than 36 in. is required. They can even be employed to compress air to 70 lb. per square inch or over. A single wheel can increase the pressure in the ratio of 1·5 to 1, so that two wheels working in series would give a pressure of 2·25, a third 3·4, and a fourth 5 atmospheres. Their mechanical efficiency is slightly inferior to that of ordinary piston compressors, but superior to Roots' blowing machines, whose efficiencies are not more than 35 to 40 per cent. For the supply of air to a blast furnace whose capacity is 160 tons of cast iron per day, and requiring 19,200 cubic feet of air per minute at atmospheric pressure, and compressed to half an atmosphere, the fan would be 2 ft. 7½ in. diameter, would run at 6,000 revolutions per minute, and the steam turbine would be about the same size. The efficiency of the turbine fan for this high power—500 useful horse power—would reach 10 per cent, corresponding to a steam consumption of 49·8 lb. of steam per useful horse power hour if the turbine worked with condensation.

CHAPTER XV.

29. *The Theory of Propeller Ventilating Fans.*—The simplest form of fan is the propeller; it requires neither diffuser nor volute, although frequently provided with a chimney; and although sometimes used for ventilating mines, its real place is in buildings where a large volume of air is required at a very low pressure—*i.e.*, when volumetric efficiency becomes far more important than manometric or mechanical. Its complete theory, however, is extremely complicated, mainly because each particle of air does not keep to a cylinder concentric with the axis of the fan. In the following approximate theory we shall suppose that it does so, and that the axial component of inflow is the same as that of outflow. In fig. 94 is shown the section of a blade, A B; the axis G H; θ is the relative angle of inflow, and ϕ

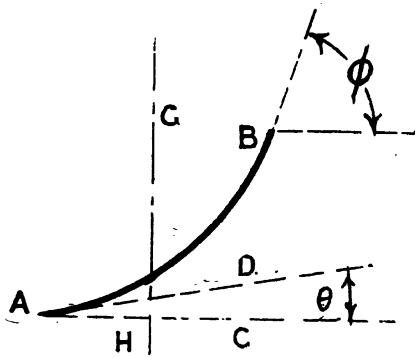


FIG. 94.

that of outflow, if we assume that the angle of flow coincides with the angle of vane. Let u be the axial component of the velocities of inflow and outflow, c the peripheral velocity at a radius r, c_1 that at the extreme radius r_1, w the component of the absolute velocity of the air at outflow at radius r, perpendicular to both radius and axis, or, in other words, the velocity of the whirl. The motion of the blade is to the left. Then

$$w = c - u \cot \phi$$

$\frac{c\,w}{g}$ = work done by the blade at that radius per pound of air, and if the air enters the wheel without sudden change of direction,

$$\cot \theta = \frac{c}{u}.$$

Under any circumstances the losses of head are

$$L_1 = \frac{(c - u \cot \theta)^2}{2\,g} \text{ at inflow,}$$

$$L_2 = \frac{w^2 + u^2}{2\,g}$$ at outflow, if no expanding chimney is used, or diffuser of any type or guide vanes which would gradually destroy w. If, however, a chimney is used, and neglecting the viscosity, if r_1 is the extreme radius of the fan and R that of the mouth of the chimney, then, if we may suppose the flow at the mouth of the chimney to be uniform, u is reduced to $u\frac{r_1^2}{R^2}$, and w, since the moment of momentum of each particle is unchanged, to $w\frac{r_1}{R}$. Hence L_2 gives place to

$$L_3 = \frac{w^2\frac{r_1^2}{R^2} + u^2\frac{r_1^4}{R^4}}{2\,g}$$

and there is also a loss due to friction, which is due (1) to the friction of the particles of the air against the outer cylindrical casing in which the fan works; (2) to the friction between the air and the surfaces of the vanes; (3) to the friction between the air and the sides of the chimney. We shall neglect (1), as it is relatively small; (2) is expressed by

$$L_4 = \frac{1}{2\,g} F_1\, u^2 \operatorname{cosec}^2 \phi,$$

and (3) by

$$L_5 = F_2 \frac{u^2}{2g} + F_3 \frac{w^3}{2gu}.$$

The last term on the right-hand side of the above is obtained as follows: Tangentially the motion of the air in the chimney is w; hence the force of friction tangentially is proportional to w^2, and as the motion is w, the total work wasted is proportional to w^3, and may be represented by

$$F_4 \frac{w^3}{2g},$$

which is independent of the quantity passing through the fan; hence the loss of energy per pound, or the loss of head, is represented by

$$F_3 \frac{w^3}{2gu},$$

because u is proportional to the weight passing through the fan.

Let us now suppose that the axial velocity is the same at any radius; then

$$Q = u \pi r_1^2,$$

and the total loss of head at inflow is

$$l_1 = \frac{\int_{r_2}^{r_1} \frac{(c - u \cot \theta)^2}{2g} 2 \pi r u \, dr}{u \pi (r_1^2 - r_2^2)}$$

where r_2 is the internal radius. It must be remembered that θ is a function of r, and not necessarily a constant. The loss of head at outflow is

$$l_2 = \frac{\int_{r_2}^{r_1} \frac{(c - u \cot \phi)^2}{2g} 2 \pi r u \, dr}{u \pi (r_1^2 - r_2^2)} + \frac{u^2}{2g}$$

where ϕ is a function of r, and $c = r\,\omega$ where ω is the angular velocity in radians. If there is a chimney, the loss of head at outflow becomes

$$l_3 = \frac{\int_{r_2}^{r_1} \frac{(c - u\cot\phi)^2}{2g} \frac{r^2}{R^2} 2\pi r u\, dr}{u\pi (r_1^2 - r_2^2)} + \frac{u^2 r_1^4}{2g R^4}.$$

The total loss of head by friction in the fan becomes

$$l_4 = \frac{\int_{r_2}^{r_1} \frac{1}{2g} F_1 u^2 \operatorname{cosec}^2\phi\, 2\pi r u\, dr}{u\pi (r_1^2 - r_2^2)}.$$

$$= \frac{F_1 u^2}{2g} \frac{\int_{o}^{r_1} 2\pi r \operatorname{cosec}^2\phi\, dr}{\pi (r_1^2 - r_2^2)}$$

and the total loss of head in the chimney is

$$l_5 = F_2 \frac{u^2}{2g} + \frac{\int_{r_2}^{r_1} F_3 \frac{(c - u\cot\phi)^2}{2g} 2\pi r\, dr}{u\pi (r_1^2 - r_2^2)}$$

Let us first suppose that the blade of the fan is a plane surface, and that $\theta = \phi$ constant. Then l_1 becomes

$$l_1 = \frac{1}{2g}\left[\frac{\omega^2 (r_1^2 + r_2^2)}{2} + u^2\cot^2\phi - \frac{4}{3}\frac{(r_1^3 - r_2^3)}{r_1^2 - r_2^2} u\,\omega\cot\phi\right]$$

To simplify this, put $r_2 = 0$.

$$l_1 = \frac{1}{2g}\left[\frac{r_1^2\,\omega^2}{2} + u^2\cot^2\phi - \frac{4}{3} r_1 u\,\omega\cot\phi\right].$$

$$= \frac{1}{2g}\left[\frac{c_1^2}{2} + u^2\cot^2\phi - \frac{4}{3} c_1 u\cot\phi\right]$$

The loss at outflow, if no chimney is used, is

$$l_2 = \frac{1}{2g}\left(\frac{c_1^2}{2} + u^2 \cot^2 \phi - \tfrac{4}{3} c_1 u \cot \phi\right) + \frac{u^2}{2g};$$

but if a chimney is used, it becomes

$$l_3 = \frac{r_1^2}{2g R^2}\left(\frac{c_1^2}{2} + u^2 \cot^2 \phi - \tfrac{4}{3} c_1 u \cot \phi\right) + \frac{u^2 r_1^4}{2g R^4}$$

The work done by the fan, per pound, is

$$\frac{\int_{r_2}^{r_1} \frac{c(c - u \cot \phi)}{g} 2 \pi u r \, d r}{u \pi (r_1^2 - r_2^2)}$$

$$= \frac{2}{g} \frac{\int_{r_2}^{r_1} (r^3 \omega^2 - u r^2 \omega \cot \phi) \, d r}{r_1^2 - r_2^2}$$

$$= \frac{2}{g}\left(\frac{r_1^2 \omega^2}{4} - \frac{u \omega r_1}{3} \cot \phi\right)$$

$$= \frac{\tfrac{1}{2} c_1^2 - \tfrac{2}{3} c_1 u \cot \phi}{g} \text{ neglecting } r_2.$$

Hence, if there is no chimney, we have

$$2g \mathrm{H} = c_1^2 - \tfrac{4}{3} c_1 u \cot \phi - (c_1^2 + 2 u^2 \cot^2 \phi - \tfrac{8}{3} c_1 u \cot \phi)$$
$$- u^2 - \mathrm{F}_1 u^2 \operatorname{cosec}^2 \phi - \mathrm{F}_2 u^2$$
$$- \text{ a term we shall neglect.}$$

We neglect the second part of l_5, as it is probably of very little importance, and the equation becomes,

$$2g \mathrm{H} = \tfrac{4}{3} c_1 u \cot \phi - u^2 (1 + 2 \cot^2 \phi + \mathrm{F}_1 \operatorname{cosec}^2 \phi + \mathrm{F}_2)$$

As this type of fan usually discharges against very little pressure, let us first suppose H = 0, and find suitable values of F_1, F_2 from experiments made by Mr. Walker with a propeller fan whose blades were plane, and set at various angles.* When the blades were set at 40 deg. to a plane perpendicular to the axis, the volumetric efficiency was 69·7, and when at 25 deg. it was 53·8 per cent. Substituting these values in the above equation—*i.e.*, putting

$$H = O, \phi = 40, \frac{Q}{c_1 r_1^2} = \frac{u \pi r_1^2}{c_1 r_1^2} = \frac{u \pi}{c_1} = \cdot 697$$

$$\text{and } \phi = 25, \frac{u \pi}{c_1} = \cdot 538, \text{ we get}$$

$$F_1 = \cdot 99, \text{ and } F_2 = \cdot 95,$$

and the equation becomes

$$\tfrac{4}{3} c_1 \cot \phi = u (1 + 2 \cot^2 \phi + \cdot 99 \operatorname{cosec}^2 \phi + \cdot 95)$$
$$= u (2 \cdot 94 + 2 \cdot 99 \cot^2 \phi).$$

The following table gives a comparison of the volumetric efficiencies from experiment and the above equation (see Table III. in Mr. Walker's paper, fan No. 16).

ϕ	15°	20°	25°	30°	35°	40°
Calculated vol. efficiency per cent	35·2	44·1	53·8	60·7	66·4	69·7
Experimental vol. efficiency per cent	28·6	43·0	53·8	64·7	68·4	69·7

It is also interesting to note that with fans 1 and 12, both of which have plane blades, the calculated volumetric efficiencies are 39·2 and 56·9, as compared with 38·2 and 58·9 respectively by experiment.

Fan No. 17 in the same paper has rounded backs to its vanes, but the calculations we have made seem to indicate that it obeys the equation

$$\tfrac{4}{3} c_1 \cot \phi = u (1 + 2 \cot^2 \phi + F_1 \operatorname{cosec}^2 \phi + F_2)$$

when $H = O, F_1 = \cdot 4625$, and $F_2 = 1 \cdot 650$;

so that $\tfrac{4}{3} c_1 \cot \phi = u (3 \cdot 06 + 2 \cdot 46 \cot^2 \phi)$.

Calculations assuming that θ and ϕ are the mean angles between backs and faces give F_1 negative.

* Proceedings of the Institution of Mechanical Engineers, 1897, "Propeller Ventilating Fans."

The following table gives the actual and calculated volumetric efficiencies :—

ϕ	15°	25°	30°	45°	50°	60°
Actual vol. efficiency per cent	41·4	60·8	68·9	76·7	75·2	40·7
Calculated vol. efficiency per cent	41·8	62·5	68·9	75	73·25	62·3

It will be seen that the agreement is remarkably close except in the last case, where the loss of head due to the friction caused by the whirling motion of the air becomes of importance, and as we have neglected it, theory cannot be expected to agree with practice.

The only efficiency that we can consider in this case is the dynamic efficiency

$$= \frac{u^2}{c_1^2\left(1 - \frac{4}{3}\frac{u}{c_1}\cot\phi\right)}$$

which can be transformed to

$$\epsilon = \frac{V^2}{\pi^2\left(1 - \frac{4}{3}\frac{V}{\pi}\cot\phi\right)}$$

where V is the volumetric efficiency.

The results obtained from this formula differ entirely from those given by experiment, because the angle of flow does not coincide with the vane angle. The above equation gives us

$$\cot\phi = \frac{1 - \frac{V^2}{\epsilon\,\pi^2}}{\frac{4}{3}\frac{V}{\pi}}.$$

The following table gives the calculated values of ϕ, showing that the angle of flow relatively to the wheel differs considerably from the vane angle.

Vane angle	15°	30°	45°	50°
ϕ	$10\frac{1}{2}$°	18·31°	23°	23·75°
Experimental efficiency, per cent	33	41·6	26·4	20·9

This difference is all the more extraordinary because one would naturally expect the curved back of the vanes to have some effect on the direction of flow.

In the experiments with fan No. 16, with plane blades, the mechanical efficiency of the fan alone is not given, but it can be calculated by deducting ·0338 horse power from that of the motor, which gives the shaft or brake horse power, and the horse power in the air divided by this gives us the efficiency of the fan alone. Applying the above formula for the mean relative angle of outflow ϕ, we obtain the following table :—

Angle of vane	15°	20°	25°	27°	30°	35°	40°
Experimental efficiency per cent of fan alone..	30·8	46·1	46·0	42·4	40·2	33·7	29·3
Volumetric efficiency per cent	28·6	43	53·8	58·0	64·7	68·4	69·7
Cot ϕ	8·01	5·26	4·06	3·72	3·26	2·95	2·81
ϕ	7°–7′	10°–46′	13°–50′	15°–3′	17°–3′	18°–44′	19°–35′

It will be seen that ϕ is half the vane angle very nearly, and it will also be noticed that the efficiency of fan No. 16 alone with plane blades is better than that with plane faces and rounded backs, No. 17, but that the volumetric efficiency is less, but not by any appreciable quantity for angles of 30 deg., 35 deg., and 40 deg.

Considering next fans with curved vanes in which θ and ϕ are constant but unequal, we find that in the equation for volumetric efficiency it is best to take θ as the angle of the face, and ϕ as the mean between the angles of the face and back. This is probably due to the fact that the outflow is affected by back and front, and is produced by a gradual change of direction in which both back and front participate, but at inflow a sudden change takes place from an axial motion to a forward, and this is produced by the front of the blade. There are three fans in Mr. Walker's paper with curved blades, Nos. 9, 14, and 15; the first and third have curved faces and backs, while the second has a plane face, but a curved back. If α is the mean angle of inclination of a blade in the first,

$$\phi = \alpha + 30\tfrac{1}{4},$$
$$\theta = \alpha - 20;$$

in the second $\phi = a + 20,$
$\theta = a;$

in the third $\phi = a + 31\frac{3}{4},$
$\theta = a - 20.$

Now, in this type of fan we have the equation

$$2gH = c_1^2 - \tfrac{4}{3}c_1 u \cot\phi - \left(\frac{c_1^2}{2} + u^2\cot^2\theta - \tfrac{4}{3}c_1 u\cot\theta\right)$$
$$- \left(\frac{c_1^2}{2} + u^2\cot^2\phi - \tfrac{4}{3}c_1 u\cot\phi\right) - (1 + F_2)u^2$$
$$- F_1 u^2 \operatorname{cosec}^2\phi$$
$$2gH = \tfrac{4}{3}c_1 u\cot\theta - u^2(\cot^2\theta + \cot^2\phi + 1 + F_2$$
$$+ F_1 \operatorname{cosec}^2\phi)$$

When H = O, this becomes

$$\tfrac{4}{3}c_1\cot\theta = u(\cot^2\theta + \cot^2\phi + 1 + F_2 + F_1\operatorname{cosec}^2\phi)$$

and the volumetric efficiency

$$\frac{u\pi}{c_1} = \frac{4\pi\cot\theta}{3(\cot^2\theta + \cot^2\phi + 1 + F_2 + F_1\operatorname{cosec}^2\phi)},$$

but the above will not give results agreeing with practice for a constant value of F_1, because, the vane being curved, the relative velocity of the air over its surface is variable. Hence, for the last term in the denominator we substitute $\frac{1}{2}F_1(\operatorname{cosec}^2\phi + \operatorname{cosec}^2\theta)$, and, as the following table shows, we obtain very good results. These three fans were tested with mean angles of 17 deg., 27 deg., and 40 deg., and we think that the results obtained with the two first angles are of very little use except to show that the last is better, both for mechanical and volumetric efficiencies, because the air at inflow was struck by the back of the blade; in fact, with a decrease in the angle of outflow, the angle of inflow is decreased too rapidly by turning round the blade. Calculating the volumetric efficiency from the equation

$$\frac{u\pi}{c_1} = \frac{4\pi\cot\theta}{3\left\{\cot^2\theta + \cot^2\phi + 1 + F_2 + \frac{1}{2}F_1(\operatorname{cosec}^2\theta + \operatorname{cosec}^2\phi)\right\}},$$

and writing $F_1 = \cdot 99$, $F_2 = \cdot 95$, as in the first case,

$$\frac{u\,\pi}{c_1} = \frac{4\,\pi \cot\theta}{3\left\{(\operatorname{cosec}^2\theta + \operatorname{cosec}^2\phi) \times 1\cdot 495 - \cdot 05\right\}}$$

which gives us the following table :—

No. of fan.	a	Actual vol. efficiency A.	Calculated vol. efficiency C.	$\frac{C}{A}$
9	40°	·80	·802	1
14	40°	·86	·902	1·05
15	40	·867	·80	·92
17	40°	·744	·75	1

The mean value of $\frac{C}{A}$ is ·995; in fan No. 17 ϕ is $a + 19$, and $\theta = a$.

We shall next consider the mean angle of outflow from Nos. 9, 14, 15, calculated from the equation

$$\cot\phi = \frac{1 - \dfrac{V^2}{\epsilon\,\pi^2}}{\frac{4}{3}\dfrac{V}{\pi}}$$

taking, of course, the actual volumetric efficiencies. This gives us the following table :—

No. of fan	9	14	15
Mean angle of vane at outflow..	70° 15′	60° 0′	71° 45′
ϕ	25° 47′	24° 15′	24° 21′

which shows that curving both faces or the back of the blade increases ϕ considerably.

To use such fans as these without a chimney to produce pressure, and to expect a high efficiency, would be absurd.

For plane blades, applying the equations

$$2\,g\,H = \tfrac{4}{3}\,c_1\,u\cot\phi - u^2\,(2\cdot 94 + 2\cdot 99\cot^2\phi),$$

$$\eta = \frac{2\,g\,H}{c_1^2\,(1 - \frac{4}{3}\frac{u}{c_1}\cot\phi)}$$

ϕ being the angle of the vane, gives us the following table :—

$\frac{u_1}{\sqrt{gH}}$	·1	·2	·3	·5	·6
η for $\phi = 15°$, per cent	9·24	18·8	19·6	10·3	....
η for $\phi = 30°$, per cent	....	7·6	12	14·5	13·85
$\frac{c_1}{\sqrt{gH}}$ for $\phi = 15°$	4·91	3·8	4·025	5·82	....
$\frac{c_1}{\sqrt{gH}}$ for $\phi = 30°$	....	5·36	4·44	4·32	4·56

showing that the maximum efficiency in the one case is near 19·6, and in the other 14·5, and that the efficiency decreases as ϕ increases.

We shall next consider the case of the above types of propeller with a chimney whose outlet has a diameter three times that of the fan. The general equation of the fan is then

$$2gH = c_1^2 - \tfrac{4}{3}c_1 u \cot\phi - \left(\frac{c_1^2}{2} + u^2\cot^2\theta - \tfrac{4}{3}c_1 u\cot\theta\right)$$
$$- \frac{r_1^2}{R^2}\left(\frac{c_1^2}{2} + u^2\cot^2\phi - \tfrac{4}{3}c_1 u\cot\phi\right) - \frac{u^2 r_1^4}{R^4}$$
$$- u^2 F_2 - \tfrac{1}{2}F_1 u^2(\operatorname{cosec}^2\phi + \operatorname{cosec}^2\theta),$$

which merely states that the head produced is equal to the work per pound done by the wheel, less the losses at inflow to the fan, outflow from the chimney, and those due to friction. This becomes, when simplified,

$$2gH = \frac{c_1^2}{2}\left(1 - \frac{r_1^2}{R^2}\right) + c_1 u\left[\tfrac{4}{3}\cot\theta - \tfrac{4}{3}\cot\phi\left(1 - \frac{r_1^2}{R^2}\right)\right]$$
$$- u^2\left[\cot^2\theta + \frac{r_1^2}{R^2}\cot^2\phi + \frac{r_1^4}{R^4} + F_2 + \frac{F_1}{2}\right.$$
$$\left.(\operatorname{cosec}^2\phi + \operatorname{cosec}^2\theta)\right]$$

and when $\frac{r}{R} = \frac{1}{3}$ this gives us

$$2gH = \tfrac{4}{9}c_1^2 + \tfrac{4}{3}c_1 u\left[\cot\theta - \tfrac{8}{9}\cot\phi\right]$$
$$- u^2\left[\cot^2\theta + \tfrac{1}{9}\cot^2\phi + \tfrac{1}{81} + F_2 + \frac{F_1}{2}(\operatorname{cosec}^2\theta + \operatorname{cosec}^2\phi)\right]$$

Let us first consider the case of plane blades when H = 0; we then get, putting F_2 = ·95 and F_1 = ·99,

$$\tfrac{4}{9} c_1^2 + \tfrac{4}{27} c_1 u \cot \phi$$
$$- u^2 \left[\tfrac{10}{9} \cot^2 \phi + \cdot 962 + \cdot 99 \operatorname{cosec}^2 \phi\right] = 0$$

$$\tfrac{4}{9} c_1^2 + \cdot 148\, c_1 u \cot \phi - u^2 \left[2\cdot 1 \cot^2 \phi + 1\cdot 952\right] = 0;$$

so that the formula for the volumetric efficiency is

$$\frac{u\pi}{c_1} = \frac{8\pi}{9\left(\sqrt{3\cdot 76 \cot^2 \phi + 3\cdot 47} - \cdot 148 \cot \phi\right)} = V,$$

which gives the following table :—

ϕ =	30 deg.	45 deg.	60 deg.	90 deg.
V per cent =	77·6	109·5	133·2	149·5

The dynamic efficiency

$$\epsilon = \frac{V}{\pi^2\left(1 - \tfrac{4}{3}\frac{V}{\pi} \cot \phi\right)}$$

and we shall here suppose ϕ to be the vane angle.

ϕ =	30 deg.	45 deg.	60 deg.	90 deg.
ϵ per cent =	14·2	26·3	26·8	22·8

We should probably get in practice a very much higher efficiency than this, as ϕ would actually be about half the above values; in the third case above, for 60 deg., supposing the angle of outflow actually 45 deg., we should have an efficiency of 41 6 per cent.

When H is not zero, we get

$$2gH = \tfrac{4}{9} c_1^2 + \cdot 148\, c_1 u \cot \phi - u_2 \left[2\cdot 1 \cot^2 \phi + 1\cdot 952\right] = 0,$$

and
$$\eta = \frac{2gH}{c_1^2\left(1 - \tfrac{4}{3}\frac{u}{c} \cot \phi\right)}$$

The following tables give velocities and efficiencies for various values of

$$\frac{u}{\sqrt{g\,H}}.$$

$\frac{u}{\sqrt{g\,H}}$	$\frac{c_1}{\sqrt{g\,H}}$ for $\phi = 15$.	$\frac{c_1}{\sqrt{g\,H}}$ for $\phi = 30$.	$\frac{c_1}{\sqrt{g\,H}}$ for $\phi = 45$.	$\frac{c_1}{\sqrt{g\,H}}$ for $\phi = 60°$.
·1	....	2·14	2·12	....
·2	2·58	2·24	2·17	2·14
·3	3·102	....	2·25	2·21
·4	....	2·64	....	....
·5	4·39	2·89	2·52	2·40

$\frac{u}{\sqrt{g\,H}}$	η for $\phi = 15$.	η for $\phi = 30$.	η for $\phi = 45$.	η for $\phi = 60°$.
·1	....	·49	·475	....
·2	·492	·502	·485	·471
·3	·435	....	·481	·457
·4	....	·442	....	....
·5	·289	·40	·429	·415

which shows that the best angle of inclination ϕ of the vanes, if we consider volumetric efficiency as well as mechanical, is about 45 deg., but that this type of fan is very unsuitable, because of its low manometric and mechanical efficiencies, for producing pressure.

We shall next consider the case of vanes in which ϕ is constant, but θ varies, so as to do away with the loss of shock at inflow; in this case the general equation becomes

$$2\,g\,H = c_1^2 - \tfrac{4}{3}\,c_1\,u\cot\phi - \frac{r_1^2}{R^2}\left(\frac{c_1^2}{2} - u^2\cot^2\phi - \tfrac{4}{3}\,c_1\cot\phi\right)$$
$$- \frac{u^2\,r_1^4}{R^4} - u^2\,F_2 - \tfrac{1}{2}\,F_1\,u^2\,(\operatorname{cosec}^2\phi + \operatorname{cosec}^2\theta),$$

taking $\operatorname{cosec}^2\theta$ as its value at $\frac{2}{3}\,r_1$, so that

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = \left(1 + \tfrac{4}{9}\,\frac{c_1^2}{u^2}\right),$$

and F_1, F_2 the previous values of ·99 and 95, the equation becomes

$$2\,g\,H = \cdot 725\,c_1^2 - 1\cdot 185\,c_1\,u\cot\phi$$
$$- u^2\,(1\cdot 952 + \cdot 606\cot^2\phi) = 0,$$

which gives the following values of peripheral speed and air efficiency in the following tables :—

$\frac{u}{\sqrt{g\,H}}$	$\frac{c_1}{\sqrt{g\,H}}$ for				
	ϕ = 30.	ϕ = 45.	ϕ = 60.	ϕ = 90.	ϕ = 135°.
·1	1·82	....	....	....	....
·2	2·03	1·87	1·79	1·69	1·53
·3	2·27	2·01	1·88	1·73	....
·5	2·84	2·36	2·12	1 82	1·47

$\frac{u}{\sqrt{g\,H}}$	η for				
	ϕ = 30°.	ϕ = 45.	ϕ = 60°.	ϕ = 90°.	ϕ = 135.
·1	·692	....	....	....	....
·2	·630	·667	·684	·70	·73
·3	·557	·616	·646	·666	....
·5	·417	·496	·529	·585	·637

Now, the volumetric efficiency is $\frac{u\,\pi}{c_1}$ for these fans, and in a Rateau fan we have

$$\text{volumetric efficiency} = \frac{Q}{c_1\,r_1^2} = \frac{u_1\,2\,\pi\,r_1\,b_1}{c_1\,r_1^2}$$

$$= \frac{u_1}{c_1},$$

since $2\,\pi\,r_1\,b_1 = r_1^2$.

Hence, even if we assume for the Rateau fan a manometric efficiency of 90 per cent, the volumetric efficiencies for $u = \cdot 5\sqrt{g}\,H$ of the two fans are 107 and 47·6 per cent, so that the propeller fan is the superior of the two in this respect, but, of course, much inferior in mechanical and manometric efficiencies. The above assumes $\phi = 135$ deg. in both cases, and even if $\phi = 90$ for the propeller, the volumetric efficiency is 86·2 per cent. No propeller fan with $\phi = 135$ has yet been constructed as far as we know, and it is quite possible that such a fan might discharge the air in the wrong direction.

CHAPTER XVI.

30. *Helical Propellers.*—We shall next consider the case of helical blades, in which, however, the pitch at inflow is not the same as at outflow, but that θ at every radius is so arranged that inflow takes place without shock, and therefore l_1 is zero. Let P be the pitch at outflow, then

$$\cot \phi = \frac{2 \pi r}{P};$$

$$\therefore \cot \phi = \frac{r}{r_1} \cot \phi_1 = A r;$$

l_3 then becomes

$$l_3 = \frac{\int_{r_2}^{r_1} \frac{(r \omega - u A r)^2}{g} \frac{r^3}{R^2} d r}{(r_1^2 - r_2^2)} + \frac{u r_1^4}{2 g R^4}$$

$$= \frac{(\omega - A u)^2}{g R^2} \frac{\int_{r_2}^{r_1} r^5 dr}{r_1^2 - r_2^2} + \frac{u r_1^4}{2 g R^4}$$

$$= \frac{(\omega - A u)^2}{6 g R^2} (r_1^4 + r_2^2 r_1^2 + r_2^4) + \frac{u^2 r_1^4}{2 g R^4}$$

$$l_4 = \frac{\int_{r_2}^{r_1} \frac{1}{2g} F_1 u^2 \frac{1}{2} (\operatorname{cosec}^2 \phi + \operatorname{cosec}^2 \theta) 2 \pi r u \, d r}{u \pi (r_1^2 - r_2^2)}$$

$$= \frac{F_1 u^2}{2 g} \frac{\int_{r_2}^{r_1} (\operatorname{cosec}^2 \phi + \operatorname{cosec}^2 \theta) r \, d r}{(r_1^2 - r_2^2)}$$

$$= \frac{F_1 u^2}{2 g} \frac{\int_{r_2}^{r_1} \left(2 + A^2 r^2 + \frac{c^2}{u^2}\right) r \, d r}{(r_1^2 - r_2^2)}$$

$$= \frac{F_1 u^2}{2 g} \frac{\int_{r_2}^{r_1} \left(2 r + A^2 r^3 + \frac{r^3 \omega^2}{u^2}\right) d r}{(r_1^2 - r_2^2)}$$

$$= \frac{F_1 u^2}{2 g} \frac{\left\{ (r_1^2 - r_2^2) + A^2 \frac{(r_1^4 - r_2^4)}{4} + \frac{\omega^2}{u^2} \frac{r_1^4 - r_2^4}{4} \right\}}{(r_1^2 - r_2^2)}$$

$$= \frac{F_1 u^2}{2 g} \left\{ 1 + \left(\frac{A^2}{4} + \frac{\omega^2}{4 u^2}\right) \left(r_1^2 + r_2^2\right) \right\}$$

and we shall take the loss of head in the chimney as $F_2 \frac{u^2}{2 g}$.

The work done by the propeller per pound is

$$\frac{\int_{r_2}^{r_1} \frac{c (c - u \cot \phi)}{g} 2 \pi u r \, d r}{u \pi (r_1^2 - r_2^2)}$$

$$= \frac{\int_{r_2}^{r_1} \frac{2 (r^2 \omega^2 - r u \pi A r)}{g} r \, d r}{(r_1^2 - r_2^2)}$$

$$\frac{1}{2 g} (\omega^2 - u A \omega) (r_1^2 + r_2^2),$$

so that the general equation of this type of fan becomes

$$H = \frac{1}{2g}(\omega^2 - u A \omega)(r_1^2 + r_2^2)$$
$$- \frac{(\omega - A u)^2}{6 g R^2}(r_1^4 + r_1^2 r_2^2 + r_2^4) - \frac{u^2 r_1^4}{2 g R^4}$$
$$- \frac{F_1 u^2}{2g}\left\{1 + \tfrac{1}{4}\left(A^2 + \frac{\omega^2}{u^2}\right)(r_1^2 + r_2^2)\right\}$$
$$- \frac{F_2 u^2}{2g}.$$

Let $r_2 = m r_1$, then

$$2 g H = (1 + m^2)(c_1^2 - c_1 u \cot \phi_1)$$
$$- \frac{(c_1 - u \cot \phi)^2 (1 + m^2 + m^4) r_1^2}{3 R^2}$$
$$- F_1 u^2 \left\{1 + \frac{(1 + m^2)}{4}\left(\cot^2 \phi_1 + \frac{c_1^2}{u^2}\right)\right\} - F_2 u^2$$
$$= c_1^2 \left\{1 + m^2 - \frac{r^2}{3 R^2}(1 + m^2 + m^4) - F_1 \frac{(1 + m^2)}{4}\right\}$$
$$- c_1 u \cot \phi_1 \left\{(1 + m^2) - \tfrac{2}{3}\frac{r_1^2}{R^2}(1 + m^2 + m^4)\right\}$$
$$- u^2 \left\{\frac{r_1^2}{3 R^2}(1 + m^2 + m^4)\cot^2 \phi_1 \right.$$
$$\left. + F_1\left(1 + \frac{(1 + m^2)}{4}\cot^2 \phi_1\right) + \frac{r_1^4}{R^4} + F_2\right\}$$

Putting $F_1 = \cdot 99$, $F_2 = \cdot 95$, $\frac{r_1}{R} = \frac{1}{3}$, this becomes

$$\{\cdot 715 (1 + m^2) - \cdot 037 m^4\} c_1^2$$
$$- \{\cdot 926 (1 + m^2) - \cdot 074 m^4\} c_1 u \cot \phi_1$$
$$- u^2 \{\cdot 285 (1 + m^2)\cot^2 \phi_1 + 1\cdot 95 + \cdot 037 m^4 \cot^2 \phi_1\}$$
$$= 2 g H.$$

Putting $m = \frac{1}{3}$, this becomes

$$\cdot 795\, c_1^2 - 1\cdot 03\, c_1\, u \cot \phi_1 - u^2 (1\cdot 95 + \cdot 3175 \cot^2 \phi_1) = 2\, g\, \text{H},$$

and
$$\eta = \frac{2\, g\, \text{H}}{(1 + m^2)(c_1^2 - c_1\, u \cot \phi_1)}$$
$$= \frac{1\cdot 8\, g\, \text{H}}{(c_1^2\, c_1\, u \cot \phi_1)}.$$

$\frac{u}{\sqrt{g\text{H}}}$	$\frac{c_1}{\sqrt{g\text{H}}}$ for $\phi = 45$.	η for $\phi = 45$.	$\frac{c_1}{\sqrt{g\text{H}}}$ for $\phi = 90°$.	η for $\phi = 90°$.
·2	1·75	·660	1·61	·691
·5	2·15	·506	1·77	·577

The above table shows that the efficiency decreases as the discharge increases, and increases with the angle ϕ. It is probably even greater for $\phi = 135$, but as the air might be discharged in the wrong direction with this arrangement, we have not considered it.

Suppose next that $\text{H} = 0$.

$$\frac{(r_1^2 - r_2^2)}{r_1^2} \frac{u\,\pi}{c} = \text{volumetric efficiency,}$$

$$= \frac{2\,\pi \times \frac{8}{9}}{1\cdot 295 \cot \phi_1 + \sqrt{11\cdot 475\; 1\cdot 596 \cot^2 \phi_1}},$$

which obviously increases with ϕ_1, and is 114 per cent for $\phi_1 = 45$ deg., and 165·5 per cent for $\phi = 90$ deg. The dynamic efficiencies calculated from the formula

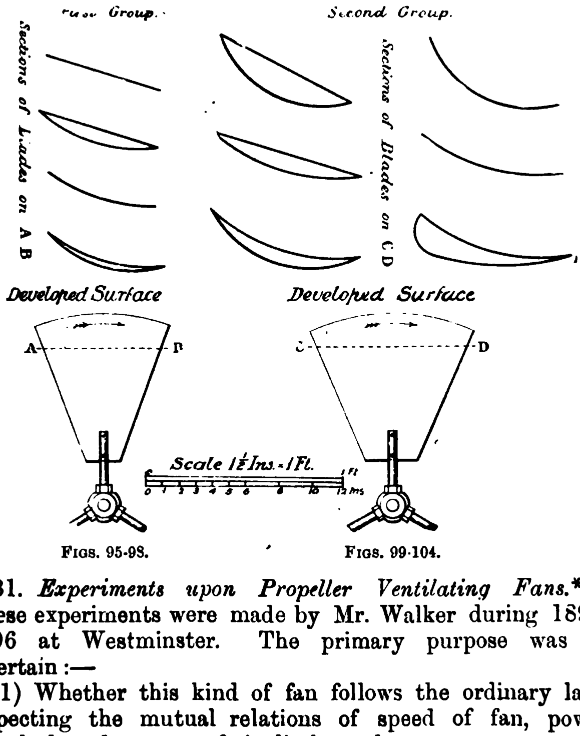
$$\epsilon = \cdot 9 \times \frac{81}{64} \cdot \frac{\text{V}^2}{\pi^2 \left(1 - \frac{9}{8}\frac{\text{V}}{\pi} \cot \phi\right)}$$

give us in these cases 24·9 per cent and 31·5 per cent, which shows an inferiority in the former, but a superiority in the latter case, so that we come to the final conclusion that, if an efficient chimney is used, ϕ_1 should be 90 deg., and the inflow edge of the blade should be helical, and so calculated that if θ_1 is the value of θ when $r = r_1$,

$$\cot \theta_1 = \frac{c_1}{u},$$

so that inflow may take place without shock, and a fair mechanical and a high volumetric efficiency can be obtained.

FIGS. 95-98. FIGS. 99-101. FIGS. 102-104.



FIGS. 95-98. FIGS. 99-104.

31. *Experiments upon Propeller Ventilating Fans.**— These experiments were made by Mr. Walker during 1895-1896 at Westminster. The primary purpose was to ascertain :—

(1) Whether this kind of fan follows the ordinary laws respecting the mutual relations of speed of fan, power absorbed, and amount of air discharged.

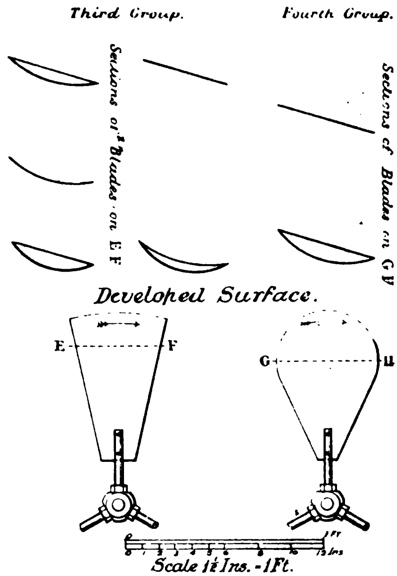
* By Mr. W. G. Walker, M.I.M.E. See Proceedings of the Institution of Mechanical Engineers, 1897.

(2) The general characteristics regarding the speed of fan, power absorbed, and quantity of air discharged, with different angles of the blades.

(3) The effect of fans differing from one another only in the cross-section of their blades.

The experiments showed that the ordinary laws are obeyed; and Mr. Walker recognises that the propeller fan is adapted to the discharge of large volumes at small pressures,

FIGS. 105-107. FIGS. 108, 109. FIGS. 110, 111.



FIGS. 105-109. FIGS. 110, 111.

and that volumetric efficiency is more important than even mechanical efficiency. All the experiments are with fans having a free discharge (with exception of a few at the end), the outlet being the same as the inlet. The volumetric efficiency of the propeller fan is greatest with free discharge, and falls off rapidly if the discharge pipe is baffled. Seventeen three-bladed fans were tested, all 23¾ in. diameter.

They are shown in figs. 95—111. The fans were driven by an electro-motor fixed centrally to a cast-iron frame in the rear of the fans, figs. 112, 113, which were keyed to the spindle

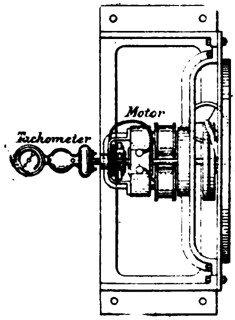


FIG. 112.

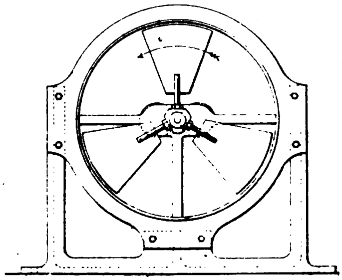


FIG. 113.

of the armature, and were thus driven at the same speed as the motor. This was a continuous-current series-wound machine of about one-third of an electrical horse power.

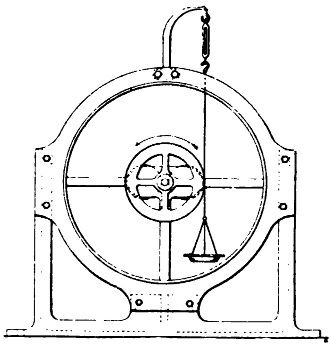


FIG. 114.

The current was taken off the 100 volt mains of the Electric Supply Corporation. The air was delivered through a tube 24 in. bore and 4 ft. long, figs. 115, 116, made of stiff sheet iron and placed concentric with the fan axis and at the end of the frame. The speed of the fan was indicated by a tachometer, attached by a hook joint to the motor spindle, and was read to two revolutions per minute. The speed of

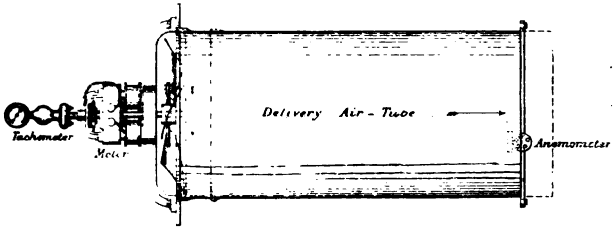


FIG. 115.

the motor was varied by electrical resistances. In most of the experiments the fans were run at 600 revolutions per minute; the speed being kept constant by the adjustment of a suitable form of resistance. The velocity of the air was measured by an anemometer of $2\frac{3}{4}$ in. diameter, placed at the outer end of the delivery tube, figs. 115, 116. The velocity

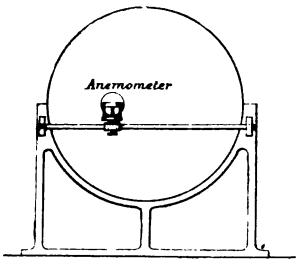


FIG. 116.

varied greatly in different positions of the same cross section of the tube. A smooth brass rod $\frac{5}{16}$ in. in diameter, was placed horizontally across the end of the tube to which the anemometer was attached, so that it could be tried at different positions of the rod. The centre of the anemometer moved in the horizontal diameter of the tube for all positions on the rod, and the instrument always moved in the same plane across the current. The B.H.P. of the motor was obtained by a dynamometric brake, fig. 114. The fan having been removed, the brake pulley was fixed in the same position upon the spindle. The brake was highly effective; it was sensitive, yet ran steady with no oscillation. The pulley was of cast iron, 9·4 in diameter, with smooth circumference; it ran true, and a fine silk cord was wound round it, the lower end attached to the scale pan, and the upper to a Salter's balance hung above the pulley and perpendicularly over the scale pan. The difference between the pull on the balance and the weight in the scale pan, multiplied by the circumference of the pulley and the revolutions per minute, and divided by 33,000, gave the B.H.P. The silk was in direct contact with the pulley. The anemometer was calibrated at Kew Observatory for speeds from 500 to 2,000 ft. per minute.

For each experiment anemometer readings were taken at each of the four following radii of the delivery tube: $1\frac{7}{8}$, $5\frac{1}{8}$, $7\frac{7}{8}$, $10\frac{5}{8}$ in. The cross section of the delivery tube was divided into four imaginary concentric rings, and each of the above radii corresponded with the centre line of one of the rings; each of the three outer rings was equal in breadth to the diameter of the anemometer. The velocity of the air in feet per minute as ascertained at each of the four radii was multiplied by the area of the corresponding rings in square feet, and the products being added together gave the number of cubic feet of air discharged per minute. The velocities given at each of the four radii are given in Tables XIX. and XX. for all the fans tried. The areas of the four imaginary rings were 1·275, ·945, ·614, ·307 sq. ft. The mean velocity of the air was obtained by dividing the air discharge in cubic feet by 3·141 sq. ft. Readings of the anemometer were taken for two minutes at each of the four radii for each experiment, together with volts, ampères, height of barometer in inches, and temperature of air. The electric motor was calibrated, so that by simply reading the amperes and noting the number of revolutions the B.H.P. was obtained. A series of experiments were made with the motor running at

TABLE XIX.—Experiments on Propeller Ventilating Fans, Figs. Tube 24 in. bore, except those marked * Revolving $4\frac{1}{2}$ in. Revolution. Anemometer placed at Radii $1\frac{7}{8}$ in., $5\frac{1}{8}$ in., $7\frac{7}{8}$ in.,

No. of fan.	Angle of blade in degs.	Revs. per minute.	Volts of motor.	Amperes of motor.	Velocity of air per minute at radius.			
					$1\frac{7}{8}$	$5\frac{1}{8}$	$7\frac{7}{8}$	$10\frac{5}{8}$
1	17	890	69·7	1·50	762	767	688	607
2	17	860	81·5	1·92	880	917	920	930
3	17	645	60·0	1·58	690	730	690	675
4	17	525	54·5	1·58	617	713	730	645
5	27	610	80·0	2·33	723	804	913	951
6	27	490	78·3	2·53	423	690	880	903
7	17	758	80·8	2·07	703	785	805	760
8	17	650	78	2·28	645	780	805	700
	27	*600	85	2·45	910	1,115	1,145	1,050
	27	595	77	2·20	670	780	875	990
	40	*495	100	2·70	735	970	1,147	1,070
9	17	600	69	1·91	320	365	530	635
	27	600	79	2·23	600	650	755	825
	27	*600	86	2·47	740	890	1,030	960
	40	570	88	2·75	745	810	910	960
10	27	600	81	2·31	805	930	950	930
	17	*600	68	1·85	260	405	555	645
11	17	850	74	1·70	640	745	850	990
	27	610	63	1·36	645	725	860	930
	27	*600	64	1·75	765	895	870	850
12	27	600	63	1·70	590	635	740	745
13	27	600	74	2·06	655	745	860	995
	17	600	61	1·63	430	525	640	680
14	17	604	53	1·40	415	540	605	640
	40	605	85	2·40	780	940	1,110	1,105
15	40	600	84	2·40	775	965	1,120	1,085
16	35	600	65	1·77	720	780	860	830
17	35	600	67	1·87	685	830	920	875

95 TO 111, OF 23¾ IN. DIAMETER, ALL REVOLVING INSIDE DELIVERY AIR OUT OF THE TUBE. BLADES SET AT AN ANGLE TO THE PLANE OF 10⅝ IN. FROM THE AXIS OF THE TUBE.

Cubic feet of air per minute.	Weight on the brake pulley in ounces.	B.H.P. of motor.	H.P. in air.	Efficiencies per cent.		
				Mechanical.	Volumetric.	Pressure.
2,128	11·0	·0458	·00[illegible]7	21·2	38·2	·7
2,888	21·5	·0866	·0241	28·0	54·0	1·4
2,172	14·2	·0430	·0103	23·9	53·0	1·4
2,139	14·2	·0350	·0098	28·0	65·0	2·1
2,790	29·0	·0829	·0218	26·3	73·0	2·7
2,536	33·5	·0770	·0164	21·3	82·0	3·4
2,427	18·5	·0650	·0144	22·2	51·0	1·3
2,330	22 2	·0677	·0127	18·8	57·0	1·6
*3,385	31·7	·0894	·0390	43·0	89·8	4·1
2,773	26·6	·0742	·0214	29·9	74·0	2·8
*3,265	39·4	·0916	·0350	38·2	105·0	5·6
1,633	20·6	·0590	·0043	7·4	43·3	·9
2,348	27·3	·0768	·0130	16 9	62·0	2·0
*2,965	32·3	·0892	·0262	29·4	78·0	3·1
2,870	37·7	·1075	·0237	22·0	80·0	3·2
*2,891	28·7	·0809	·0243	30·0	76·7	3 0
*1,675	19·7	·0550	·0047	8·5	44·4	1·0
2,720	15 5	·0620	·0202	32·6	50·9	1·3
2,641	16·0	·0458	·0185	40 4	68·9	2·4
*2,699	17 6	·0489	·0190	40·4	71·6	2·6
2,222	16·8	·0473	·0110	23·3	58·9	1·7
2,737	23·8	·0671	·0200	30·0	72·6	2·6
1,926	15 3	·0431	·0070	16·7	51·0	1·3
1,846	10·7	·0300	·0063	21·0	48·6	1·2
3,272	31·0	·0874	·0348	39·8	86 0	3·7
3,270	30·6	·0862	·0350	40·9	86·7	3·8
2,580	18·1	·0510	·0172	33·7	68·4	2·4
2,705	19·1	·0538	·0199	37·0	71·7	2·6

TABLE XX.—EXPERIMENTS ON A PROPELLER VENTILATING FAN, NO. 17, WITH BLADES SET AT DIFFERENT ANGLES TO THE PLANE OF REVOLUTION. FAN DIAMETER, 23¾ IN., REVOLVING IN TUBE, 24 IN. DIAMETER; SPEED, 600 REVOLUTIONS PER MINUTE, ANEMOMETER PLACED AT RADII OF 1⅞, 5⅛, 7⅞, 10⅝ IN; SEE FIG. 107.

Angle of blades.	Motor.		Velocity of air in feet per minute at radius.				Cubic feet of air per minute.	Horse power.		Efficiencies.	
	Volts.	Ampères.	1⅞	5⅛	7⅞	10⅝		Brake.	Air.	Mechanical.	Volumetric.
Deg											
15	40	1·05	460	470	510	510	1,562	·0115	·0038	33·0	41·4
20	45	1·22	570	590	610	630	1,916	·0181	·0070	38·5	50·8
25	52	1·36	630	630	730	770	2,291	·0282	·0120	42·2	60·8
27	55	1·43	640	710	790	820	2 423	·0334	·0143	42·8	64 3
30	65	1·66	665	770	875	860	2,598	·0423	·0176	41·6	68·9
35	67	1·87	685	830	920	875	2,705	·0538	·0199	37·0	71·7
40	76	2·13	705	835	970	900	2,805	·0707	·0222	31·4	74 4
45	84	2·40	780	910	960	880	2,827	·0858	·0227	26·4	76·7
50	91	2·59	790	880	930	870	2,771	·1024	·0214	20·9	75·2
60	86	2·48	210	290	450	680	1,534	·0911	·0035	3·2	40·7

600 revolutions per minute, and the experimental readings are shown in fig. 117. The straightness of the current and torque lines shows that a relation between torque and current of the form

$$t = a i - b,$$

where t = torque and i = current, and a and b are constants, must hold good.

It is obvious how this diagram enables the B.H.P. at 600 revolutions to be calculated. With a given torque the amperes were not quite constant for all speeds of the motor;

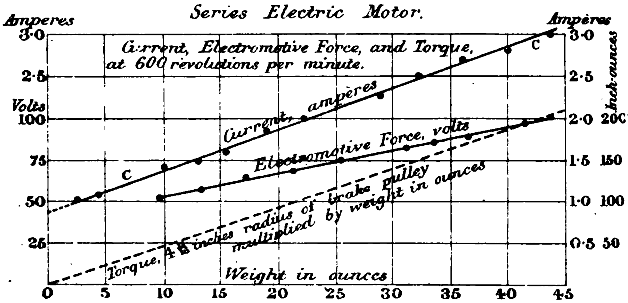


FIG. 117.

they increased slightly and uniformly with the increase in the number of revolutions per minute. It was easy, however, to frame a formula which gave the torque at any speed the motor might be running at, taking into account the small increase of the current due to speed. Although not essential for the present experiments, it was interesting to determine at what speeds the motor should be run so as to give the maximum B.H.P. and maximum efficiency severally. Its speed characteristics are shown in fig. 118 for constant E.M.F.; the revolutions are plotted as abscissæ, and the ordinates are electrical horse power—*i.e.*, volt-amperes ÷ 746. The diagram is self-explanatory. Fans 1 to 6 were tried at progressive speeds from 300 to 1,000 revolutions per minute, and the following relations were verified for constant angle of blades and position of fan: (1) Air discharge varies as speed of revolution; (2) horse power varies as (speed of revolution)^3; (3) horse power varies as (discharge)^3; (4) torque varies as (speed of revolution)^2; (5) torque varies as electric current.*

The following is the method of calculating the horse power in the air discharged and the efficiencies. The weight of 1 cubic foot of air at temperature F Fahrenheit $= \frac{1\cdot3304\,B}{F + 461}$, where B is the height of the barometer in inches. Taking

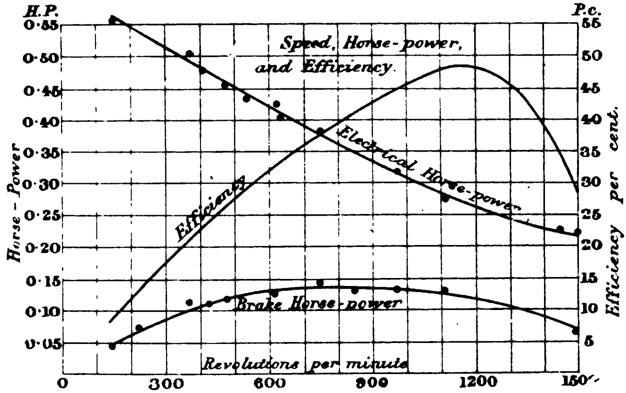


FIG. 118.

into account the moisture in the air, the weight of 1 cubic foot of air

$$= \frac{1\cdot3304}{F + 461}\,(B - \tfrac{3}{8}\,b),$$

where b is the pressure due to the moisture in inches of mercury.

If W = weight of air discharged in pounds, and V = velocity of air in feet per second, then the kinetic energy of

* Fig. 109 shows that this is not the case.

the air discharged is $\frac{W V^2}{2 g}$, and the horse power of air discharged is

$$\frac{W V^2}{2 g \times 550} = V^3 \times \text{constant},$$

for the same fan under same conditions. Hence if Q = cubic feet of air per second, the horse power of the air discharged

$$= \frac{V^2 Q}{550 \times 64{\cdot}4} \times \frac{1{\cdot}3304 B}{F + 461}$$

$$= \frac{V^2 Q B}{F + 461} \times {\cdot}00003756.$$

The mechanical efficiency $= \frac{\text{horse power in air discharged}}{\text{B.H.P.}}$, volumetric efficiency is as usual $\frac{Q}{c_1 r_1^2}$, and dynamic pressure efficiencies are evidently $\frac{V^2}{2 c_1^2}$, the static pressure efficiency being zero.

Experiments were made with fan blades at different angles to the plane of rotation. The results with fan 17, having plane surfaces and rounded backs to the blades, are given in Table XX., and plotted in fig. 119. These may be termed the characteristic curves of the fan for varying angles. It should be noted that maximum volumetric efficiency is not obtained with the same angle as maximum mechanical efficiency. In Table XXI. fans 16 and 17 have been compared, and the latter certainly has the better volumetric efficiency by a very small amount, and the mechanical efficiency of fan and motor, which, of course, includes motor friction, is better for the latter; but when we deduct ·0338 horse power for the bearing friction of the motor, the efficiencies of fan alone are as below :—

Angle of vane	15°	20°	25°	27°	30°	35°	40°
Mechanical efficiency of fan, No. 16	30·8	46·1	46·0	42·4	40·2	33·7	29·3
Mechanical efficiency of fan, No. 17	33	38·5	42·2	42·8	41·6	37	31·4

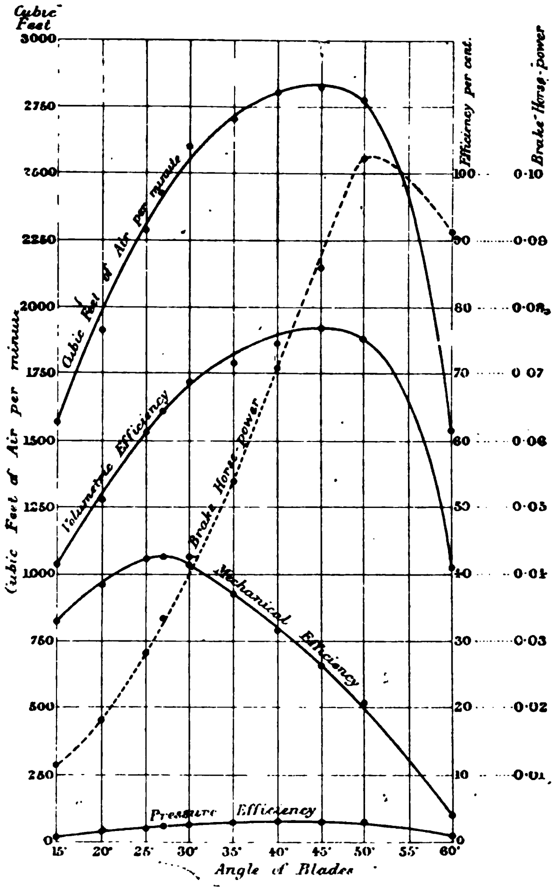

Cubic Feet
3000
2750
2500
2250
2000
1750
1500
1250
1000
750
500
250
0
Cubic Feet of Air per minute
Efficiency per cent.
Brake Horse-power
100 0·10
90 0·09
80 0·08
70 0 07
60 0·06
50 0·05
40 0·04
30 0·03
20 0·02
10 0·01
0
Cubic Feet of Air per minute
Volumetric Efficiency
Brake Horse-power
Mechanical Efficiency
Pressure Efficiency
15° 20° 25° 30° 35° 40° 45° 50° 55° 60°
Angle of Blades

Fig. 119.

TABLE XXI.—COMPARISON OF PROPELLER VENTILATING FANS NOS. 16 AND 17 WITH BLADES SET AT DIFFERENT ANGLES TO THE PLANE OF REVOLUTION. BOTH FANS ARE $23\frac{3}{4}$ IN. DIAMETER, REVOLVING INSIDE A DELIVERY AIR TUBE OF 24 IN. BORE; SPEED 600 REVOLUTIONS PER MINUTE. FRICTION OF MOTOR INCLUDED.

Angle of blades.	Air discharged in cubic feet per minute.	Horse power.		Efficiency per cent.			Air discharged in cubic feet per minute.	Horse power.		Efficiency per cent.		
		of motor = B.H.P. + H.P. required to overcome friction.	Useful horse power in the air.	Mechanical.	Volumetric.	Pressure.		of motor = B.H.P. + H.P. required to overcome friction.	Useful horse power in air.	Mechanical.	Volumetric.	Pressure.
Deg. 15	1,079	·0377	·0012	3·1	28·6	·41	1,562	·0453	·0038	8·3	41·4	·87
20	1,622	·0429	·0042	9·7	43·0	·94	1,916	·0519	·0070	13·4	50·8	1·31
25	2,030	·0521	·0084	16·1	53·8	1·48	2,291	·0620	·0120	19·3	60·8	1·88
27	2,185	·0588	·0106	18·0	58·0	1·73	2,423	·0672	·0143	21·2	64·3	2·10
30	2,439	·0698	·0145	20·7	64·7	2·13	2,598	·0761	·0176	23·1	68·9	2·42
35	2,580	·0848	·0172	20·2	68·4	2·39	2,705	·0876	·0199	22·7	71·7	2·62
40	2,627	·0958	·0182	18·9	69·7	2·47	2,805	·1045	·0222	21·2	74·4	2·82

So that one is about as good as the other. Seventeen three-bladed fans were tried in order to test the effect of the cross-section of the fan blades, fig. 95 to fig. 111. They may be divided into four groups. The first comprises 1 to 4, the second 5 to 10, the third 11 to 15, and the fourth 16 and 17. The blades were of sheet iron $\frac{1}{16}$ in. thick, and excepting 10 their cross-sections are either lines or circles. The fans in each group differed from one another only in the cross-section of their blades, which were flat, plano-convex, concavo-convex, of different curvatures. Fan 1 had flat blades. Fan 2 was formed by fixing a circular back to fan 1. Fan 3 was formed by curving the blades of fan 1. Fan 4 was formed by fixing to the back of fan 3 a still more convex surface. The blades of the other groups were similar in form, but of different area and thickness. These changes in shape produced considerable effect. Fan 1 to fan 4 were all tried with their blades at 17 deg. inclination. The superiority in mechanical and volumetric efficiencies of the last should be noticed.

Air is sucked, strange to say, into the outer circumference of the fan, and the volumetric efficiency is largely increased—see fan 8 at 40 deg.—by exposing the whole of the outer circumference of the fan. Some of the fans were tried with exposed perimeter by moving the delivery tube $4\frac{1}{2}$ in. forward, as shown dotted in fig. 115. Thus in the case of fan 9 the mechanical and volumetric efficiencies were increased from 16·9 to 29·4 and 62 to 78 per cent respectively. A much wider form of blade may be used in fans arranged to feed from the tips. This type of fan should therefore, where possible, be fixed with its circumference exposed.

Mr. Walker made experiments with two fans 24 in. and 48 in. diameter; the former was a three-bladed fan, with blades set at an angle of 35 deg. to the plane of rotation; it is a kind which has been designed and employed by Mr. Walker for the ventilation of buildings, factories, and ships, and for drying. It was tested at 600 revolutions per minute, and was driven by belt from a shunt-wound motor. Anemometer readings were taken at a distance of 18 in. in front of the fan, as well as behind. At inflow on an elliptical surface the velocity at the centre was 303 ft. per minute, at

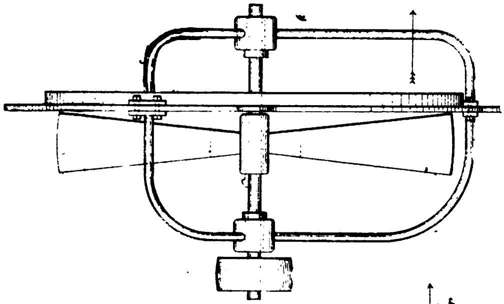


Fig. 121.

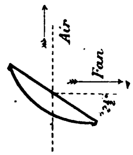


Fig. 122.

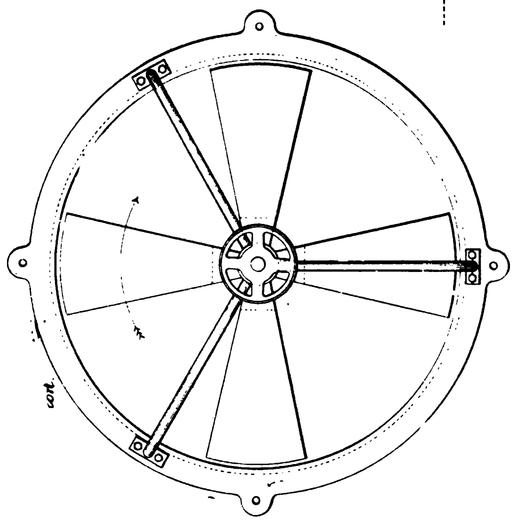


Fig. 120.

about the middle of the curve 280 ft. per minute, and on the major axis, which was perpendicular to that of the fan, it was 250 ft. per minute. The velocities of discharge were 600 at the centre, and 875, 1,230, 1,130, 470, and 175 at radii of 1·91, 5, 7·65, 10·5, and 12 in. No delivery tube was used; the air on the delivery side did not spread, as an anemometer at 13 in. radius did not move. The 48 in. fan was tested in the same manner, the blades being set at 32½ deg. to the plane of rotation. It is made for tea-drying in India, and in order to reduce shipping charges is made as light as possible. The arms carrying the fan spindle are of mild steel 1 in. in diameter, screwed into cast-iron bosses and to a cast-iron ring. The blades are hollow

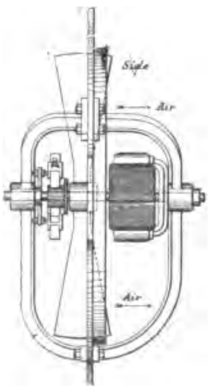


FIG. 123.

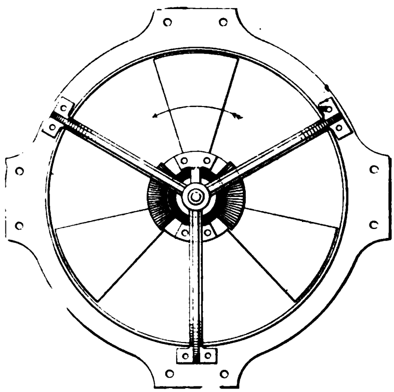


FIG. 124.

plano-convex of $\frac{1}{32}$ in. thick steel, brazed together and riveted upon the lugs of a cast-iron boss, figs. 120, 121, 122. Though so light, the blades are so rigid as to be practically incapable of vibration; consequently the fans are nearly silent at all speeds.

In this fan, when driven at 350 revolutions without a delivery tube, the velocities of inflow, measured on a half ellipsoid, were 540 near the centre, increasing to 600 at about the middle of the curve and to 830 on the diameter of the fan.

At the discharge the velocities at the centre, and $3\frac{1}{2}$ in., 10 in., 15 in., 21 in., and 24 in. radius, were 1,070 ft., 1,370 ft., 1,660 ft., 1,490 ft., 870 ft., and 400 ft. per minute respectively. Anemometer readings were taken at a distance of 3 ft. in front of the fan, and at the elliptical curve behind. The fan delivered 17,000 cubic feet of air per minute at 350 revolutions and about 34,000 cubic feet at 700 revolutions. In figs. 123 and 124 is shown a fan with electric motor specially designed for it. The field magnets are cylindrical, and are shown in section in fig. 125 ; they are made as small as possible in order to offer as little obstruction as possible

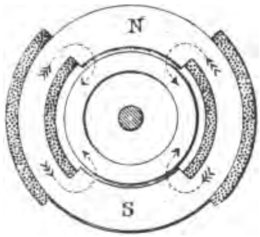


FIG. 125.

to the passage of the air. The position of motor that least obstructed the air was a different position on the spindle depending on the orifice; the best position was generally found to be a little in front of the fan. There appears to be a central region immediately in front of the fan where only a little stream of air is delivered, owing probably, in fan working with free discharge, to the centrifugal action on the front face of the blades, which is apparent near the centre.*

Several experiments were also made with contracted outlet and inlet with a fan $23\frac{3}{4}$ in. diameter, with blades set at 35 deg. to the plane of rotation, and driven at 800 revolutions to the minute by a belt from a shunt-wound motor.

The fan was placed in the same position as before, and it discharged into a 2 ft. diameter delivery tube 4 ft. long with

* It is obvious that the centre does less work per pound than the outer part, since $\frac{c\,w}{g}$ is the work per pound done by the fan.

partially closed outlet, having central holes 6 in., 12 in., and 18 in. diameter The fan was tried both for propelling and exhausting air, and its efficiency was in both cases much reduced. One of the reasons that this kind of fan was unable to maintain static pressure in the air is probably the comparatively slow speed of the blades near the centre, in consequence of which the air tended to pass back again through the centre of the fan. The effect was therefore tried of fixing a circular disc in front of the fan on the delivery side, so as to prevent the air from returning through it; this had the effect of increasing the efficiency to a great extent when working against resistance, whereby a static

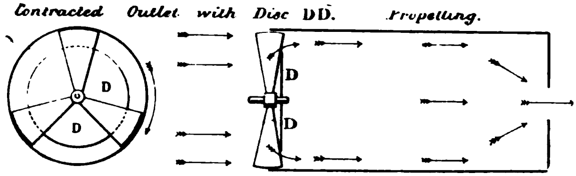


FIG. 126.

pressure was obtained in the air delivered. Experiments were made with discs of different diameters, and it was found that the size of disc should increase with the contraction of orifice to obtain a good efficiency. The reason for this is evident, because *c* and *w* are greater near the outer circumference, and *c w* must at least be equal to *g* H, and is generally much more.

Figs. 126, 127, and 128 show the fan driving the air through a contracted orifice, drawing it through the same (in both cases) with a disc, and trying to drive it through without a disc; in the last case the air to a great extent returns through the centre of the fan. Considering that this type of fan is used for drawing air through a material to be dried like wool, or through tortuous flues, as in refrigerating apparatus, the adoption of the central disc becomes a necessity. The fan is more efficient when exhausting than when producing pressure. Without a circular disc 31 cubic feet of air per minute were driven through the 6 in. orifice at 800 revolutions per minute, and the delivery was increased to 451 ft. when exhausting, the other conditions being identical; the volumetric efficiency was thus increased nearly 15 times. With the 12 in. orifice the discharges were 497 when blowing and 1,374 when exhausting. With the 18 in. orifice the

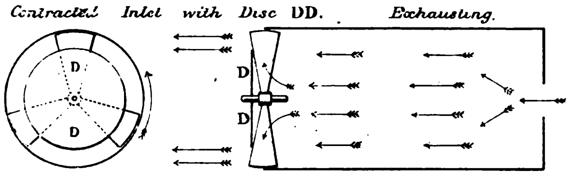


FIG. 127.

volumetric efficiency was increased only from 58½ per cent when blowing to 67 when exhausting. In the opinion of the writer the advantage of suction over blowing is largely due to the fact that the dynamic head is very much greater in the latter than in the former, because the air has to leave by a contracted orifice with considerable velocity, whereas, when exhausting, the kinetic energy of entry is at

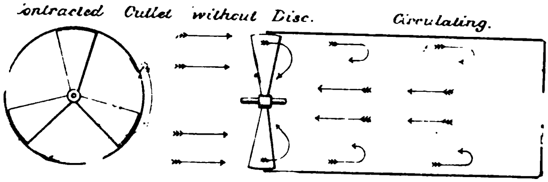


FIG. 128.

least partly converted into pressure head at outflow. Hence, when blowing, some of the air returns through the fan, as the fan is incapable of producing the necessary pressure to give it the velocity that would carry it through the small outlet, while, when exhausting, as the outlet is large, the pressure, and therefore the work required, is much less. To take an illustration. Allowing a coefficient of contraction of ·65, the head in feet required to discharge 31 cubic feet of air through an orifice 6 in. diameter is given by

$$Q = c A \sqrt{2 g H}$$

$$H = \frac{Q^2}{2 g c^2 A^2} = \frac{31^2 \times 4^2}{3600 \times 64 \times (\cdot 65)^2 \times (\cdot 7854)^2}$$

$$= 0 \cdot 256 \text{ ft.},$$

Q being cubic feet per second, A the area of circle 6 in. diameter in square feet, and H the head in feet, while the head due to the discharge of 451 cubic feet through a 2 ft. diameter circle, with probably no contraction, is only ·0877 ft. Mr. Walker points out that negative slip has been generally noticed with propellers having thick blades and round backs. Now, his experiments show that the flow through fans with curved backs is very much greater than that through fans with plane blades, and this appears to partly explain negative slip. It will be noticed that the mean pitch of the aft edge of the propeller, considering back and face, is greater than the pitch of the face alone.

Mr. Walker also carried out some supplementary experiments with fans of six, three, and two blades, all of the same shape and set at 30 deg. to the plane of rotation. The revolutions were 600 per minute; the six-bladed fan was tried first, and its alternate blades were then removed so as to form a three-bladed fan. Afterwards the two-bladed fan was tried. The discharges were 2,350, 2,535, and 2,140 cubic feet per minute, giving volumetric efficiencies of 62, 67, and 57 per cent. The mechanical efficiency of the three-bladed fan was also the highest. Experiments were also made with helical blades of constant pitch, and considerable variation was noticed in the axial discharge of the air. The angle at the tips was 27 deg. with the plane of revolution. The helical blades were made of $\frac{1}{16}$ in. sheet brass pressed on a wood mould. The fan was run at 600 revolutions, and the anemometer was placed 18 in. in front of the fan. No delivery tube was employed, and the axial velocities at radii of 2 in., 5 in., and 12 in. were 770, 1,340, and 230, while at the centre the velocity was 665. The rotary velocities were also measured by placing the anemometer wheel in a plane passing through the axis. The velocities at radii of 2 in., 5 in., 8 in., and 12 in. were 234 ft., 562 ft., 530 ft., and 185 ft. per minute. The efficiency was increased by putting rounded backs to the blades, but the experiments showed that helical blades did not possess any advantages over the ordinary non-helical blades. With plano-convex fans of later design Mr. Walker obtained volumetric efficiencies of 86 to 90 per cent, but he does not give the angle of blade.

The horse power necessary for driving the fan to produce a given discharge of air is as follows: Taking the barometer at 30 in., the temperature of the air at 60 deg. Fah., and the mechanical efficiency at 30 per cent, let d be the diameter of the fan in feet, a the area of the fan disc in square feet, V the velocity of the air in feet per second, and Q the quantity of air discharged in cubic feet per second, then

$$\text{H.P. required to drive the fan} = \frac{\text{H.P. in discharged air}}{\text{mechanical efficiency}}.$$

$$= \frac{V^2 Q B}{T} \times \frac{\cdot 00003756}{\cdot 3},$$

and substituting $T = 60 + 461$, and $V = \frac{4\,Q}{\pi\,d^2}$

$$\text{H.P. required to drive the fan} = \frac{Q^3}{d^4} \times \cdot 0000115.$$

Taking the volumetric efficiency at 90 per cent, this for a 2 ft. diameter fan means 3,400 cubic feet at 600 revolutions per minute. Let N = revolutions per minute, then

$$N = \frac{60\,Q}{\cdot 9 \times 2\,\pi\,r_1^{\,3}} = \frac{85\,Q}{d^4}.$$

It is evident from these two formulæ that d should be as large as possible. Of course these formulæ only apply to free discharge. The effect of increase of diameter is shown by the fact that to discharge 6,000 cubic feet of air per minute the 2 ft. fan would require ·72 horse power, while a 4 ft. fan would need only ·045.

CHAPTER XVII.

OTHER PROPELLER VENTILATING FANS AND RATEAU SCREW FANS.

THERE are many types of these in use at the present day. The Hattersley-Pickard fan is shown in figs. 129 and 130. It is fitted with a boss of large diameter in the centre, because that part of the propeller fan is not only useless for

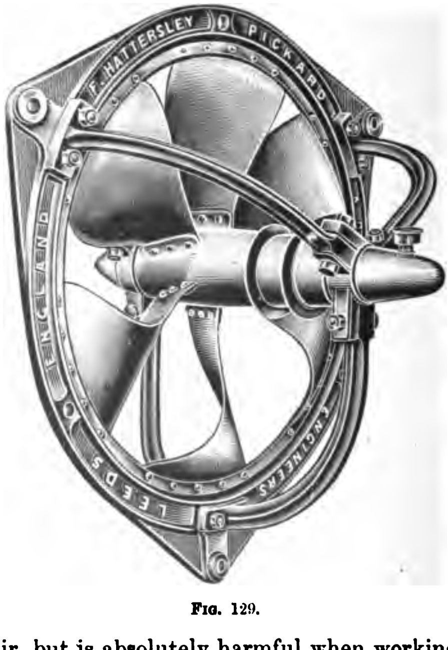


FIG. 129.

moving air, but is absolutely harmful when working against pressure, for which, as well as volume, this fan is designed. The blades are made to lean towards the intake so as to enable the air to cross them at right angles, and therefore with the least possible friction. They are of helical con-

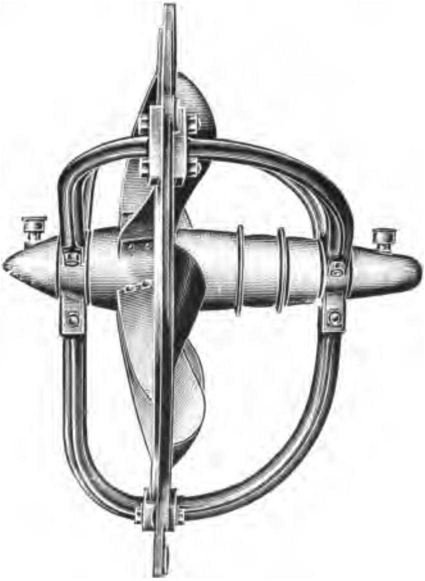


Fig. 130.

Fig. 131.

struction, with a longitudinally or axially increasing pitch—*i.e.*, ϕ is greater than θ, and at the outer circumference they appear excellently adapted for drawing in the air radially as

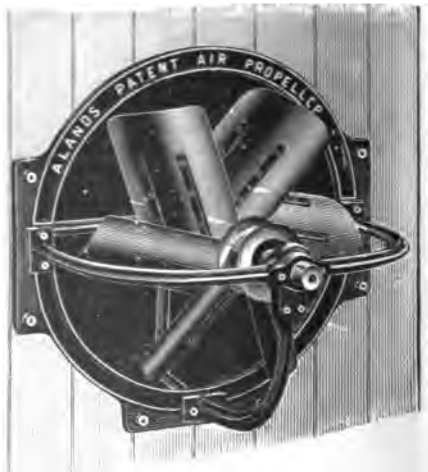


FIG. 132.

well as axially. This appears to be a most carefully designed fan. The following is a list of particulars of standard sizes :—

Diameter of blades in inches.	Revolutions per minute.	Cubic feet of air per minute.	Actual horse power required.	Diameter of pulleys in inches.	Width of belt in inches.	Volumetric efficiency at maximum discharge and revolutions.
18	700 to 1,200	2,150 to 3,600	$\frac{1}{6}$ to $\frac{3}{8}$	$3\frac{1}{4}$	$1\frac{1}{2}$	113%
24	500 to 900	3,500 to 6,600	$\frac{1}{5}$ to $\frac{5}{8}$	4	$1\frac{3}{4}$	117%
30	450 to 750	5,700 to 9,800	$\frac{1}{4}$ to 1	5	2	..
36	400 to 650	9,500 to 16,000	$\frac{1}{2}$ to $1\frac{1}{4}$	6	$2\frac{1}{2}$	..
48	300 to 550	17,500 to 31 500	1 to $2\frac{1}{4}$	8	$3\frac{1}{2}$	118%

The Blackman fan is very largely used in this country; a drawing of it is shown in fig. 131, the Aland fan with curved blades in fig. 132, and a fan constructed by the American Blower Company in fig. 133. The last is shown

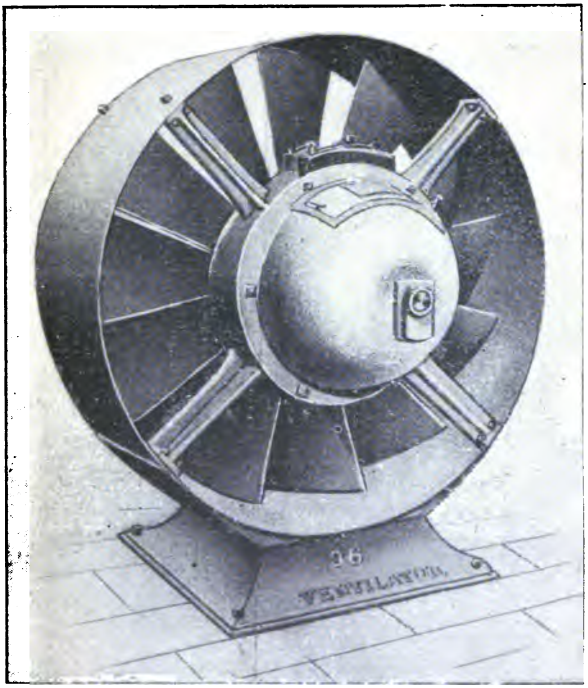


FIG. 133.

with electric motor attached. Figs. 134 and 135 show a propeller fan made by Messrs. Beck and Henkel, of Cassel. The casing is conical, and this type of fan is intended to discharge large quantities of air at low pressures. The vanes are made of steel. The rest of the construction is obvious from the drawing. These fans are made with diameters from 10 in. to 118 in., and can be driven direct or by belt.

33. *Rateau Screw Fans.*—The propeller type of fan is certainly that best suited for delivering a large amount of air at an exceedingly small pressure; but where a large

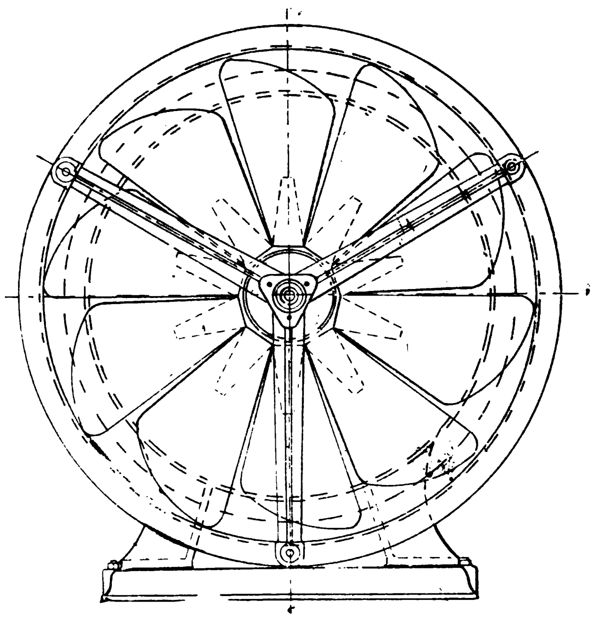


FIG. 134.

amount is required, while there is some pressure to contend with, a modification of the propeller fan, in which only the outer portions of the blades (*i.e.*, the part where c is great) are used, seems to us to be preferable; of this we have abundant proof in the experiments made by Mr. Walker with a central disc fitted to his fan. And if, in addition, the fan casing is so constructed that the tangential component of the air as it leaves the fan—namely, w—is afterwards reduced, we have every reason to expect an increase of efficiency, if this casing does not increase the friction to such an extent that it does more harm than good. We cannot get rid of w at discharge from the fan, because at any point $\frac{c\,w}{g}$ is the work per pound, and if H is not zero, w cannot be,

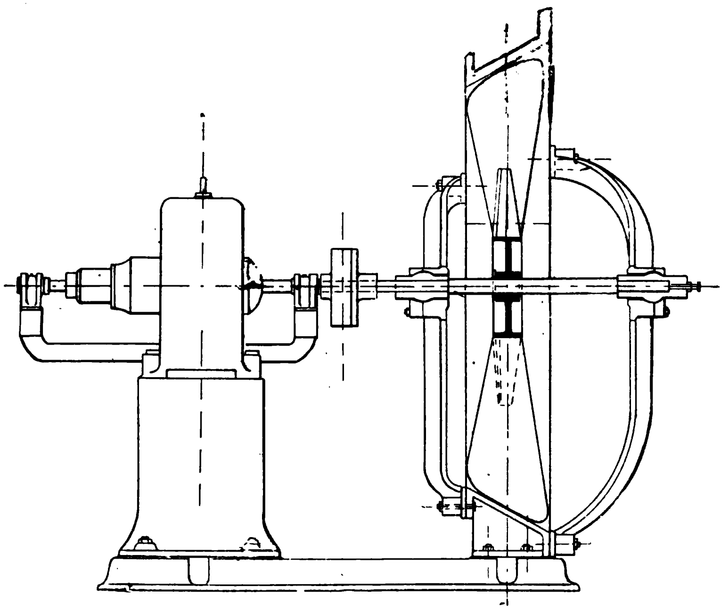


FIG. 135.

unless at inflow we give the air moment of momentum before it enters the wheel, and then in the wheel destroy this altogether, so that the air issues with axial velocity. The fan shown in figs. 136, 137, 138, does this. In the lower part of fig. 137 is shown a cylindrical section through the guide and wheel blades of a Rateau screw fan. The direction of the air's motion is shown by the arrows. The guide vanes are *m*, *m* and the wheel vanes *a*, *b*. The motion of the wheel is upwards, and the wheel vanes are so designed that inflow takes place at the normal orifice without shock, and the air is discharged axially without any whirling motion at all. The sectional elevation of fig. 137 needs no comment, except that the chamber D has its inner side partly conical in order to reduce the velocity of the air by increasing the section of discharge. The wheel, fig. 136, has its vanes

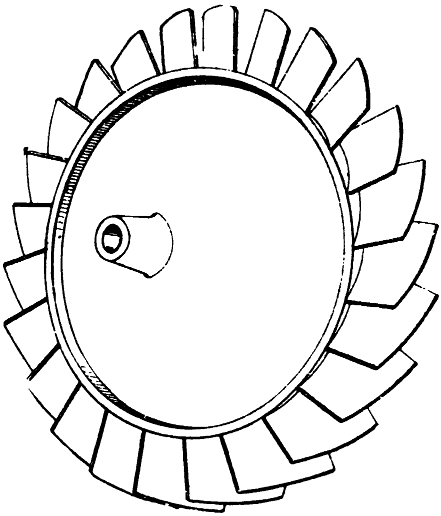


FIG. 136.

formed of steel plate fixed to the rim of a slightly conical wheel of cast iron or bronze, by means of angle irons in large sizes, or by embedding them into the rim in smaller sizes. A general view is seen in fig. 138. An alternative method is shown in figs. 139, 140. The spiral admission chamber gives the entering air velocity in the opposite direction to that of rotation, and after leaving the fan with a velocity wholly axial, the air is discharged through a passage whose section is increased by making its inner surface the frustrum of a cone. A third method is shown in fig. 141; here the air enters parallel to the axis, and the vanes at inflow are so inclined as to receive it without shock. The change of the moment

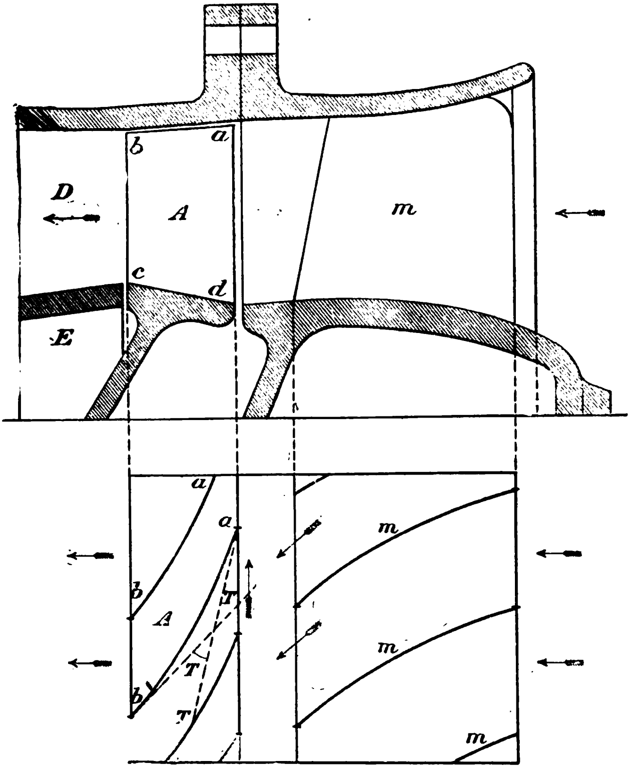


FIG. 137.

of momentum is effected by giving the air forward tangential motion at discharge, and a volute is provided to reduce this and convert its kinetic into pressure energy. A modification of the third method is shown in fig. 142. Here the air enters from the left; there are no guide vanes, so that the

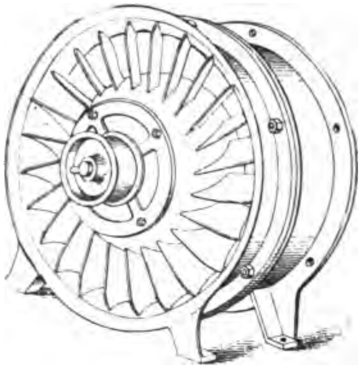


Fig. 138.

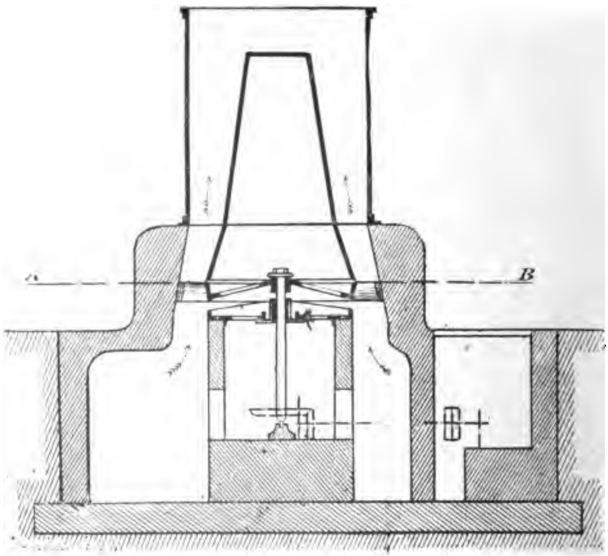


Fig. 139.

air must be discharged in a forward direction from the fan. It then enters a diffuser whose inner surface is cylindric, but whose outer is a frustrum of a cone, and these two are connected by vanes which change the forward motion of the air to an axial direction, so that the greater part of its kinetic energy is converted to pressure, A test of a ventilator of this type is given in the table below. This fan was afterwards fitted in the French warship Jena. The test was

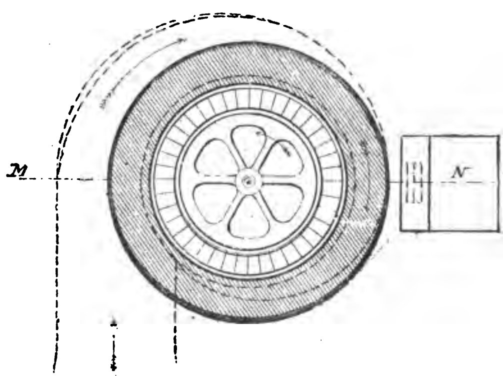


FIG. 140.

made in the workshop, the air being discharged from the fan, whose diameter was 4·69 ft., into a long tube 5·57 ft. diameter, partially closed at the further end by a steel plate, in which there was a square orifice whose area was varied, the discharge being calculated from the pressure in the tube, and the section of the orifice, a coefficient of contraction varying between ·65 and ·70 being used according to the section of the orifice; the formula for calculating the discharge per second in cubic metres being—

$$Q = c\,A\sqrt{2\,g\,H}$$

$$= c\cdot A\sqrt{\frac{2 \times 9{\cdot}81 \times h \times \text{density of water}}{1000 \times \text{density of air}}}$$

where Q = cubic metres of air per second,
A = area of section in square metres,
c = coefficient of contraction,

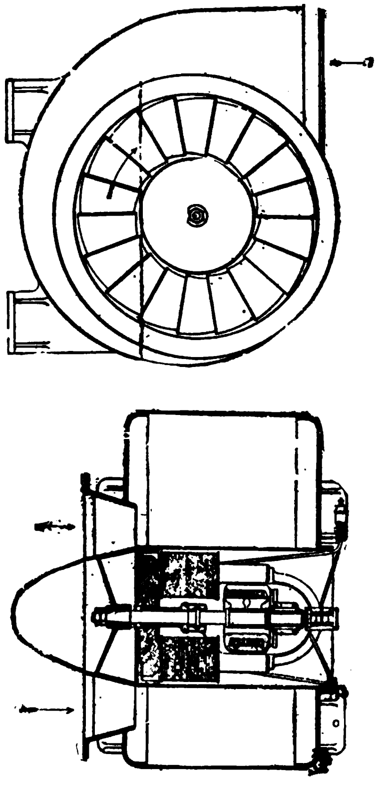


FIG. 141.

H = pressure of air in metres of air,
g = 9·81 in metric units,
h = water gauge in millimetres.

TEST OF AN AXIAL OR SCREW FAN OF THE TYPE SHOWN IN FIG. 142, of 4·69 FT. EXTREME DIAMETER OF VANES.

No. of revolutions per minute..	526	545	585	532	552	556	556	564	573
Water gauge in inches........	1·84	1·61	1·53	1·53	1·53	1·53	1·34	1·29	1·22
Kilowatts of motor	10·7	9·9	9·65	9·85	10·1	10·0	9·5	9·6	9·9
Foot-pounds of work by fan per second	0	1156	1705	2390	3030	3170	3180	3180	3461
Discharge in cubic feet per second	0	137	215	296	386	396	452	474	554
Mechanical efficiency of fan and motor	0	·16	·24	·32	·41	·43	·45	·45	·47*
Manometric efficiency	·26	·21	·205	·21	·19	·19	·18	·155	·14
Volumetric efficiency	0	·185	·30	·42	·52	·53	·61	·62	·71*

* It should be noted that maximum volumetric and mechanical efficiencies occur at the same orifice.

34. *The Theory of Rateau Screw Fans.*—We shall first consider the type shown in figs. 136, 137, and 138. Referring to the sectional view of guide and wheel vanes, fig. 137, let α be the angle made by the guide vanes m at the mean radius of the wheel, with a plane perpendicular to the axis, θ ϕ the angles made with such a plane by the wheel vanes at inflow and outflow. Let c_1 be the speed of the wheel at the mean radius and u_2 the axial component of the air at discharge from the guide wheel. Then, if the air enters the moving wheel without shock,

$$\cot\theta = \frac{c_1}{u_2} + \cot\alpha,$$

and we leave it to the reader to construct for himself the parallelogram of velocities. Again, at outflow, which is axial, if u_1 is the axial component of discharge, which in fig. 137 is greater than u_1, then

$$c_1 = u_1 \cot\phi.$$

The work done per pound of air by the wheel is

$$\frac{c_1 w_2}{g} = \frac{c_1 u_2 \cot\alpha}{g}$$

Hence, when working at the speed at which shock at inflow is avoided, the general equation of the fan is

$$H = \frac{c_1 u_2 \cot \alpha}{g} - \frac{u_3^2}{2g}(1 + F) - F_1 \frac{u_2}{2g} \operatorname{cosec}^2 \alpha$$
$$- \frac{F_2 u_2^2}{2g} \operatorname{cosec}^2 \theta - \frac{F_2 u_1^2}{2g} \operatorname{cosec}^2 \phi,$$

where u_3 is the velocity of discharge from the fan casing to the left of D, and the last two terms are introduced to represent more accurately than one the frictional loss in the wheel.

Let b_1, b_2, b_3 be the breadths of the wheel at outflow and inflow, and of the casing to the left of D, and let r_1 be the mean radius of the wheel; then, if Q = cubic feet per second,

$$Q = 2\pi r_1 b_1 u_1 = 2\pi r_1 b_2 u_2 = 2\pi r_1 b_3 u_3.$$

Neglecting the thicknesses of the vanes,

$$2gH = 2 c_1 \cot \alpha \frac{Q}{2\pi r_1 b_2} - \frac{Q^2}{4\pi^2 r_1^2 b_3^2}(1 + F)$$
$$- F_1 \frac{Q^2 \operatorname{cosec}^2 \alpha}{4\pi^2 r_1^2 b_2^2} - F_2 \frac{Q^2 \operatorname{cosec}^2 \theta}{4\pi^2 r_1^2 b_2^2}$$
$$- F_3 \frac{Q^2 \operatorname{cosec}^2 \phi}{4\pi^2 r_1^2 b_1^2},$$

or it may be put in the form—

$$2gH = \frac{2 c_1 u_1 b_1 \cot \alpha}{b_2} - \frac{u_1^2 b_1^2}{b_3^2}(1 + F)$$
$$- \frac{u_1^2 b_1^2}{b_2^2}(F_1 \operatorname{cosec}^2 \alpha + F_2 \operatorname{cosec}^2 \theta)$$
$$- F_3 u_1^2 \operatorname{cosec}^2 \phi.$$

Substituting in this, $\cot \phi = \dfrac{c_1}{u_1}$ and

$$\operatorname{cosec}^2 \theta = \operatorname{cosec}^2 \alpha + \frac{c_1^2 b_2^2}{u_1^2 b_1^2} + 2 \frac{c_1}{u_1} \cdot \frac{b_2}{b_1} \cot \alpha,$$

we obtain an equation between H, c_1, u_1, b_1, b_2, b_3, and α.

$$2\,g\,\mathrm{H} = 2\,c_1\,.\,u_1\,.\,\frac{b_1}{b_2}\,.\,\cot\alpha\,(1 - \mathrm{F}_2) - c_1^2\,(\mathrm{F}_2 + \mathrm{F}_3)$$
$$- b_1^2\,u_1^2\left(\frac{1 + \mathrm{F}}{b_2^2} + \frac{(\mathrm{F}_1 + \mathrm{F}_2)\,\mathrm{cosec}^2\,\alpha}{b_2^2} + \frac{\mathrm{F}_3}{b_1^2}\right).$$

But in designing this type of fan it is best to commence by making certain assumptions. We can safely assume that the mechanical efficiency of the fan alone is about 60 per cent, and as this type of fan is intended to give a high volumetric efficiency, we should take this as 75 per cent; then

$$\mathrm{V} = \text{volumetric efficiency} = \frac{\mathrm{Q}}{c_0\,r_0^2},$$

where c_0 = peripheral speed,

r_0 = greatest wheel diameter,

$$\mathrm{V} = \frac{2\,\pi\,r_1^2\,b_2\,u_2}{c_1\left(r_1 + \frac{b_2}{2}\right)^3}.$$

Now, we shall suppose that $b_2 = \frac{r_1}{3}$

$$\mathrm{V} = \frac{2}{3}\,\frac{\pi\,u_2}{1{\cdot}166 \times 1{\cdot}36\,c_1} = \frac{1{\cdot}317\,u_2}{c_1};$$

$$\therefore\ u_2 = \frac{\mathrm{V}c_1}{1{\cdot}317} = {\cdot}569\,c_1 \text{ if } \mathrm{V} = {\cdot}75,$$

$$\eta = \frac{g\,\mathrm{H}}{c_1\,w_2} = \frac{g\,\mathrm{H}}{c_1\,u_2\cot\alpha};$$

$$\therefore\ \cot\alpha = \frac{1{\cdot}317\,g\,\mathrm{H}}{{\cdot}6\,c_1^2\,\mathrm{V}} = \frac{2{\cdot}195\,\mathrm{M}}{\mathrm{V}}$$

and if $\mathrm{V} = {\cdot}75$

$$\cot\alpha = 2{\cdot}93\,\mathrm{M}$$

$$u_1 = \frac{b_2\,u_2}{b_1} = \frac{7}{6}\,u_2 \text{ let us say,}$$
$$= {\cdot}665\,c_1$$

and $\cot\phi = \frac{c_1}{u_1} = 1{\cdot}5$

$$\phi = 33 \text{ deg.} - 41 \text{ min.},$$

independent of the manometric efficiency, and dependent only on V, r_1, b_1, and b_2. Again,

$$\cot \theta = \frac{c_1}{u_2} + \cot \alpha = 1{\cdot}76 + 2{\cdot}93\,\text{M}.$$

The following is a list of values of α and θ for various values of M, calculated from the above equations :—

M per cent =	10	20	40	60
α = ...	73° 40′	59° 38′	40° 29	29° 38′
θ = ...	25° 58′	23° 5′	18° 50′	15° 52′

In the next type of fan with spiral inflow passage but no guide vanes, the manometric efficiency will increase the greater the velocity of air in this passage, which is also w_2, the tangential component of the velocity of inflow. Let a be the section of this passage, and Q the discharge in cubic feet per second; then

$$w_2 = \frac{\text{Q}}{a}, \qquad u_2 = \frac{\text{Q}}{2\,.\,\pi\,.\,r_1\,.\,b_2},$$

$$u_1 = \frac{\text{Q}}{2\,\pi\,r_1\,b_1}, \text{ and } c_1 = u_1 \cot \phi,$$

$$\cot \theta = \frac{c_1 + w_2}{u_2}.$$

Then we may proceed as follows :—

$$\eta = \frac{g\,\text{H}}{c_1\,w_2};\ w_2 = \frac{g\,\text{H}}{c_1\,\eta}.$$

$$\frac{w_2}{\sqrt{g\,\text{H}}} = \frac{\sqrt{\text{M}}}{\eta} = k \text{ say},$$

so that if η and M are assumed, c_1 and w_2 are known in terms of H ;

$$\cot \theta = \frac{c_1 + w_2}{{\cdot}569\,c_1}$$

and is now known, and $u_1 = {\cdot}665\,c_1$ as before, assuming

$$\frac{b_1}{r_1}, \frac{b_2}{r_1}, \text{ and V},$$

as in the previous case. Then

$$\cot \phi = \frac{c_1}{u_1} = 1{\cdot}5, \text{ and } \phi = 33^\circ\ 41' \text{ as before.}$$

We need not deal with the type of fan in fig. 141, as its theory is precisely the same as that of the radial-flow fan with volute.

In designing the type, fig. 142, we shall suppose the volumetric efficiency 75 per cent as before, and that

$$b_1 = \frac{4}{11} r_1 \,;\ b_2 = \frac{2}{3} r_1 \,;$$

then
$$V = \frac{2\,\pi\, r_1^2\, b_2\, u_2}{c_1\left(r_1 + \frac{b_2}{2}\right)^3} = \frac{\pi\, u_2}{(1\frac{1}{3})^2\, c_1},$$

$$\frac{u_2}{c_1} = {\cdot}566\ V = {\cdot}425,$$

$$\frac{u_1}{c_1} = \frac{b_2}{b_1} \cdot \frac{u_2}{c_1} = {\cdot}777,$$

c_1 being, of course, the volocity at the mean radius;

$$\cot \theta \text{ at inflow} = \frac{c_1}{u_2} = 2{\cdot}35$$

$$\theta = 23^\circ\ 3'.$$

If ϕ is the angle made by the mean direction of motion of the air relative to the wheel at outflow, then

$$w_1 = c_1 - u_1 \cot \phi$$

$$\cot \phi = \frac{c_1 - w_1}{u_1} = 1{\cdot}286\left(1 - \frac{w_1}{c_1}\right).$$

Assuming
$$\eta = {\cdot}6,\ w_1 = \frac{g\,H}{{\cdot}6\, c_1}$$

$$\frac{w_1}{c_1} = 1{\cdot}66\ M\,;$$

$$\therefore \cot \phi = 1{\cdot}286\ (1 - 1{\cdot}66\,M).$$

In the diffuser the guide vanes must not suddenly alter the direction of motion of the air, which has an axial component u_1 and a tangential w_1 if these vanes receive it

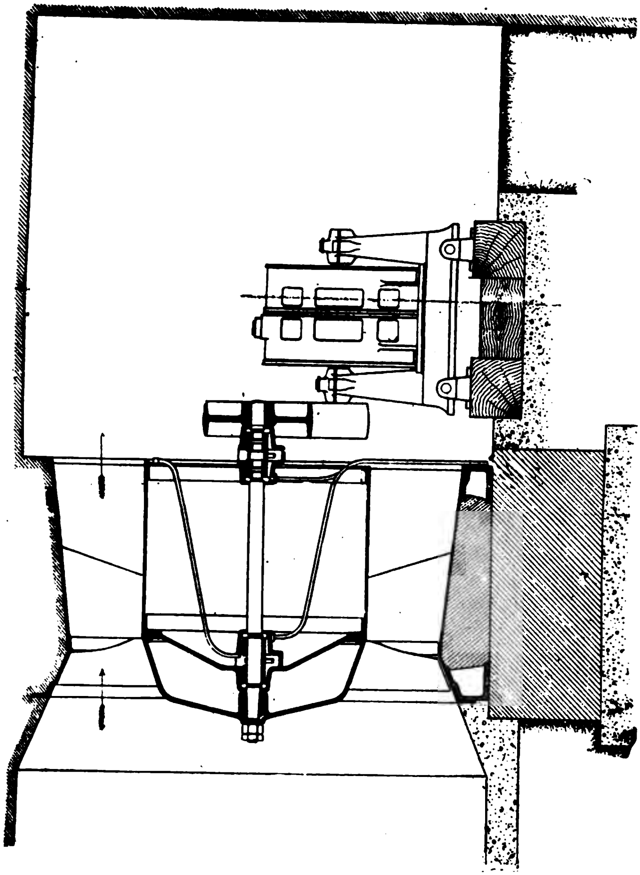


Fig. 142.

immediately after it leaves the wheel; if a is the angle they make with the plane of revolution,

$$\tan a = \frac{u_1}{w_1} = \cdot 777 \frac{c_1}{w_1}$$

$$= \frac{\cdot 466}{M}.$$

The following is a list of values of ϕ and a for various values of M :—

M per cent =	10	20	30	40	60	80
ϕ = ...	43° 4′	49° 21′	57° 8′	66° 38′	68° 54′	86° 7′
a = ...	77° 52′	66° 46′	57° 15′	49° 21′	37° 49′	30°

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